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Wave-induced current for long-crested and short-crested random waves

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ABSTRACT

This paper provides a simple analytical tool which can be used to calculate the wave-induced current beneath long-crested (2D) and short-crested (3D) random waves. The approach is based on assuming the waves to be a stationary narrow-band random process and by adopting the Forristall (2000) wave crest height distribution representing both 2D and 3D Stokes second order random waves. An example is included to illustrate the applicability of the results for practical purposes using data typical for field conditions; the significant values of the Stokes drift and transport in deep water and in finite water depth are calculated. The present analytical results can be used to make assessment of the wave-induced current based on available wave statistics.

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1. Introduction

The Stokes drift represents an important transport mechanism in the ocean, which locally are responsible for material tracer evolution (e.g. plankton, larvae, contaminated ballast water from ships, oil spills). It is also involved in air-sea mixing processes across the interphase between the atmosphere and the ocean. The Stokes drift is obtained as the mean Lagrangian velocity giving the water particle drift in the wave propagation direction. This drift has its maximum at the surface and decreases towards the bottom. The total mean mass transport is obtained as the integral over the water depth of the Stokes drift; this is also referred to as the volume Stokes transport by Rascle et al. (2008). More details of the Stokes drift are given in Dean and Dalrymple (1984).

The Stokes drift and the volume Stokes transport are commonly defined for regular waves. However, their characteristic quantities are also defined for random waves in terms of the sea state parameters significant wave height and characteristic wave periods (e.g Rascle et al. (2008); Webb and Fox-Kemper (2011)). Rascle et al. (2008) described a global data base for parameters associated with ocean surface mixing and drift, which included the surface Stokes drift and the volume Stokes transport among other parameters by performing wave hindcast of the wave parameters. Rascle and Ardhuin (2013) improved the hindcast results of Rascle et al. (2008) by using new parameterizations of the physical processes involved (more details are given in the references therein). Webb and Fox-Kemper (2011) considered relationships between the wave spectral moments and the Stokes drift in deep water at an arbitrary elevation in the water column, and intercomparisons were made using different spectral formulations. Myrhaug (2013, in press) presented bivariate distributions of significant wave height with surface Stokes drift and volume Stokes transport. Myrhaug (2013) also presented bivariate distributions of spectral peak period with these two Stokes drift parameters together with example of results corresponding to typical field conditions.

The purpose of this study is to provide a simple analytical tool which can be used to give estimates of the significant value of the wave-induced current, i.e. the Stokes drift as well as the Stokes transport, within a sea state of long-crested (2D) and short-crested (3D) Stokes second order random waves. The approach is based on assuming the waves to be a stationary narrow-band random process and adopting the Forristall (2000) wave crest height distribution representing both 2D and 3D random waves. The cumulative distribution function of Stokes drift and Stokes transport for individual random waves are determined, from which the statistical properties of both quantities can be calculated. Thus this approach is more mathematically sound than by using characteristic statistical values of the waves in the regular wave formulas. An example is also included to illustrate the applicability of the results for practical purposes using data typical for field conditions.

2. Background for regular waves

Following Dean and Dalrymple (1984) the mean (time-averaged) Lagrangian mass transport at an elevation z_1 in the water

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column in finite water depth h is given as

$$\overline{u}_L = \frac{ga^2k^2}{\omega}\frac{\cosh 2k(z_1+h)}{\sinh 2kh} \tag{1}$$

Here, g is the acceleration due to gravity, a is the linear wave amplitude, k is the wave number corresponding to the cyclic wave frequency ω given by the dispersion relationship $\omega^2 = gk\tanh kh$. Eq. (1) indicates that the water particles drift in the wave propagation direction; this drift has its maximum at the mean free surface $z_1 = 0$ and decreases towards the bottom as $z_1 \to -h$. In deep water Eq. (1) reduces to

$$\overline{u}_L = \frac{ga^2k^2}{\omega}e^{2kz_1}; \quad \omega^2 = gk \tag{2}$$

The Langrangian mass transport is often referred to as Stokes drift.

The total mean (time- and depth-averaged) mass transport is given as (Dean and Dalrymple, 1984)

$$M = \frac{\rho g a^2 k}{2\omega} \tag{3}$$

where ρ is the density of the fluid. M is often referred to as the Stokes transport. More details of the Stokes drift and the Stokes transport are given in Dean and Dalrymple (1984).

3. Present analytical calculation of wave-induced drift in random waves

At a fixed point in a sea state with stationary narrow-band random waves consistent with Stokes second order regular waves in finite water depth, the non-dimensional nonlinear crest height, $w_c = \eta_c/a_{rms}$ is

$$w_c = \hat{a} + O(k_p a_{rms}) \tag{4}$$

Here $\hat{a} = a/a_{rms}$ is the non-dimensional linear wave amplitude, where the linear wave amplitude a is made dimensionless with the root-mean-square (rms) value a_{rms}. Moreover, $O(k_p a_{rms})$ denotes the second order (nonlinear) terms, which are proportional to the characteristic wave steepness of the sea state, $k_p a_{rms}$, where k_p is the wave number corresponding to ω_p (=peak frequency of the wave spectrum) given by the dispersion relationship for linear waves (which is also valid for the Stokes second order waves):

$$\omega_p^2 = gk_p \tanh k_p h \tag{5}$$

Now Eq. (4) can be inverted to give $\hat{a} = w_c - O(k_p a_{rms})$. By substituting this in Eq. (1), the non-dimensional Stokes drift for individual random waves, $u = \overline{u}_L/u_{Lrms}$, is given as

$$u = w_c^2 \tag{6}$$

where

$$u_{Lrms} = \frac{ga_{rms}^2 k_p^2}{\omega_p}\frac{\cosh 2k_p(z_1+h)}{\sinh 2k_p h} \tag{7}$$

In deep water Eq. (7) reduces to

$$u_{Lrms} = \frac{ga_{rms}^2 k_p^2}{\omega_p}e^{2k_p z_1}; \quad \omega_p^2 = gk_p \tag{8}$$

Similarly, the non-dimensional Stokes transport for individual random waves, $m = M/M_{rms}$, is given as

$$m = w_c^2 \tag{9}$$

where

$$M_{rms} = \frac{\rho g a_{rms}^2 k_p}{2\omega_p} \tag{10}$$

It should be noted that strictly speaking, the non-dimensional Stokes drift (and the non-dimensional Stokes transport) is related to both wave crest height and wave number. For example, in deep water, $\omega^2 = gk$ and $\omega_p^2 = gk_p$, and consequently the non-dimensional Stokes transport is given by

$$m = M/M_{rms} = \left(\frac{\rho g a^2 k}{2\omega}\right) \Big/ \left(\frac{\rho g a_{rms}^2 k_p}{2\omega_p}\right) = \frac{a^2}{a_{rms}^2}\sqrt{\frac{k}{k_p}} = w_c{}^2\sqrt{\frac{k}{k_p}}.$$

However, under the assumption of narrow-band wave spectrum (i.e. $k = k_p$), the relations suggested in this paper are acceptable approximations.

Now the Forristall (2000) parametric crest height distribution based on simulations using second order theory is adopted. The simulations were based on the Sharma and Dean (1981) theory; this model includes both sum-frequency and difference-frequency effects. The simulations were made both for 2D and 3D random waves. A two-parameter Weibull distribution with the cumulative distribution function (cdf) of the form

$$P(w_c) = 1 - \exp\left[-\left(\frac{w_c}{\sqrt{8}\alpha}\right)^{\beta}\right]; \quad w_c \ge 0 \tag{11}$$

was fitted to the simulated wave data. The Weibull parameters α and β were estimated from the fit to the simulated wave data, and are based on the wave steepness S_1 and the Ursell parameter U_R defined by

$$S_1 = \frac{2\pi H_s}{g\,T_1^2} \tag{12}$$

and

$$U_R = \frac{H_s}{k_1^2 h^3} \tag{13}$$

Here H_s is the significant wave height, T_1 is the spectral mean wave period and k_1 is the wave number corresponding to T_1. The wave steepness and the Ursell number characterize the degree of nonlinearity of the waves in finite water depth. At zero steepness and zero Ursell number fits were forced to match the Rayleigh distribution, i.e. $\alpha = 1/\sqrt{8} \approx 0.3536$ and $\beta = 2$. Note that this is the case for both 2D and 3D linear waves. The resulting parameters for the 2D-model are

$$\alpha_{2D} = 0.3536 + 0.2892S_1 + 0.1060U_R$$
$$\beta_{2D} = 2 - 2.1597S_1 + 0.0968U_R^2 \tag{14}$$

and for the 3D-model

$$\alpha_{3D} = 0.3536 + 0.2568S_1 + 0.0800U_R$$
$$\beta_{3D} = 2 - 1.7912S_1 - 0.5302U_R + 0.284U_R^2 \tag{15}$$

Forristall (2000) demonstrated that the wave set-down effects were smaller for short-crested than for long-crested waves, which is due to that the second-order negative difference-frequency terms are smaller for 3D waves than for 2D waves. Consequently the wave crest heights are larger for 3D waves than for 2D waves.

The sum-frequency and difference-frequency effects arise from adding together all the frequency components which give second order terms with frequencies equal to the sum of two pair frequencies (sum-frequencies), and second order terms with frequencies equal to the difference of two pair frequencies (difference-frequencies). The terms with the sum-frequencies represent the short period second order wave components, while those with the difference-frequencies represent the long period second order wave components. The second order effects increase with decreasing water depth. The difference-frequency terms have almost no effect in deep water (for a narrow-band process it is zero). But as the water depth decreases, these terms become more significant, and are almost of the same magnitude as the

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