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Nonlinear vibration of anisotropic laminated cylindrical shells with piezoelectric fiber reinforced composite actuators

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ABSTRACT

This paper deals with the small and large amplitude flexural vibrations of anisotropic shear deformable laminated cylindrical shells with piezoelectric fiber reinforced composite (PFRC) actuators in thermal environments. Two kinds of fiber reinforced composite (FRC) laminated shells, namely, uniformly distributed and functionally graded reinforcements, are considered. The motion equations are based on a higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity and including the extension-twist, extension-flexural and flexural-twist couplings. The thermo-piezoelectric effects are also included, and the material properties of both FRCs and PFRCs are estimated through a micromechanical model and are assumed to be temperature dependent. A boundary layer theory and associated singular perturbation technique are employed to determine the linear and nonlinear frequencies of hybrid laminated cylindrical shells. The numerical illustrations concern the cross-ply and angle-ply laminated cylindrical shells with fully covered or embedded PFRC actuators under different sets of thermal and electric loading conditions. Detailed parametric studies are carried out to investigate effects of material property gradient, temperature variation, applied voltage, shell geometric parameter, stacking sequence, as well as the shell end conditions on the linear and nonlinear vibration characteristics of the hybrid laminated cylindrical shells.

1. Introduction

Piezoelectric materials are extensively used in the construction of marine and offshore structures (Chopra, 2002) in the quest for lightweight flexible structures with self-controlling and/or self-monitoring capabilities. Piezoelectric fiber reinforced composites (PFRCs) have the potential to provide various sensing functions for nondestructive testing and may be used as sensing elements for structural health monitoring and adaptive material systems (Brunner et al., 2009). By taking advantage of the direct and converse piezoelectric effects, hybrid composite structures with embedded or surface-bonded piezoelectric sensors and actuators can adapt harsh environmental condition by combining the traditional advantages of composite laminates with the inherent capability of piezoelectric materials.

Many studies have been made on linear vibration analysis and vibration control of fiber reinforced composite (FRC) laminated cylindrical shells with integrated piezoelectric sensors and actuators (Hussein and Heyliger, 1998; Ray, 2003; Vel and Baillargeon, 2005; D'Ottavio et al., 2006; Zhang et al., 2008; Alibeigloo and Kani, 2010). In the aforementioned studies, piezoelectric layers are usually placed at the extreme thickness positions of a shell-like structure to achieve the most effective actuation. However, surface-bonded actuators are likely to be damaged by contact with surrounding objects. Moreover, for large scale structural control applications such as marine structures, monolithic piezoelectric actuators and sensors suffer from certain shortcomings with regard to tailorable anisotropic actuation. PFRCs have been introduced to address these concerns. Linear vibration control and dynamic analysis of hybrid laminated cylindrical shells/panels with distributed PFRC actuators were performed by Ray and Reddy (2005) and Kapuria and Kumari (2010).

Nonlinear flexural vibration behavior of FRC cylindrical shells has received considerable attention (Lakis et al., 1998; Toorani and Lakis, 2004; Jansen, 2008). However, although in a well-known reference case there seems to be a reasonable agreement, there are unresolved discrepancies between the results obtained by different authors (Jansen, 2008). This is mainly due to the fact that the displacement assumed does not satisfy either governing equations or boundary conditions. To the best of the authors' knowledge, there is no literature covering nonlinear flexural vibration behavior of FRC cylindrical shells with piezoelectric layers. In the aforementioned studies, however, the fiber reinforcements are usually assumed to be distributed uniformly in each ply and the

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fiber volume fraction does not vary spatially at the macroscopic level.

Functionally graded materials (FGMs) are a new generation of composite materials in which the microstructural details are spatially varied through nonuniform distribution of the reinforcement phase. Two kinds of FGMs are designed to improve mechanical behavior of plate/shell structures. One is functionally graded unidirectional fibers reinforced composites (Birman, 1995; Feldman and Aboudi, 1997; Shen, 2012a, 2012b). Another one, like functionally graded ceramic–metal materials, is functionally graded particles reinforced composites (Zhao et al., 2009; Li et al., 2010; Shariyat et al., 2010; Shen, 2012c). The concept of functionally graded material can be utilized for the laminates by non-homogeneous distribution of fiber reinforcements into the matrix with a specific gradient so that the mechanical behavior of laminated shells can be improved.

The postbuckling behaviors of functionally graded (FG) FRC cylindrical shells subjected to either axial compression or lateral pressure in thermal environments were recently studied by Shen (2012a, 2012b). In the present work, we focus our attention on the nonlinear flexural vibration of functionally graded FRC laminated cylindrical shells with PFRC actuators in thermal environments. The novel contribution of the present work is that the PFRC and FG-FRC are both taken into account. The equations of motion are based on a higher order shear deformation theory with a von Kármán-type of kinematic nonlinearity and including the extension-twist, extension-flexural and flexural-twist couplings. The thermo-piezoelectric effects are also included, and the material properties of both FRCs and PFRCs are estimated through a micromechanical model and are assumed to be temperature dependent. Motion equations are first deduced to a boundary layer type, and a singular perturbation technique is then employed to determine the linear and nonlinear frequencies of hybrid laminated cylindrical shells. The numerical illustrations show the effect of material property gradient, temperature rise, shell geometric parameter, stacking sequence, and the applied voltage on the linear and nonlinear vibration characteristics of hybrid laminated cylindrical shells.

2. Effective material properties of FRCs and PFRCs

We assume that the effective material properties of FRCs and PFRCs can be expressed in terms of a micromechanical model by the rule of mixture, such that (Shen and Zhu, 2011)

$$E_{11} = V_f E_{11}^f + V_m E^m \tag{1a}$$

$$\frac{1}{E_{22}} = \frac{V_f}{E_{22}^f} + \frac{V_m}{E^m} - V_f V_m \frac{\nu_f^2 E^m / E_{22}^f + \nu_m^2 E_{22}^f / E^m - 2\nu^f \nu^m}{V_f E_{22}^f + V_m E^m} \tag{1b}$$

$$\frac{1}{G_{ij}} = \frac{V_f}{G_{ij}^f} + \frac{V_m}{G^m} \quad (ij = 12, 13 \text{ and } 23) \tag{1c}$$

$$\nu_{12} = V_f \nu^f + V_m \nu^m \tag{1d}$$

$$\rho = V_f \rho^f + V_m \rho^m \tag{1e}$$

where E_{11}^f, E_{22}^f, G_{12}^f, G_{13}^f, G_{23}^f, ν^f and ρ^f are the Young's moduli, shear moduli, Poisson's ratio and mass density, respectively, of the fiber, while E^m, G^m, ν^m and ρ^m are the corresponding properties for the matrix, respectively. V_f and V_m are the fiber and matrix volume fractions and must satisfy the unity condition of $V_f + V_m = 1$. Similarly, the effective thermal expansion coefficients in the longitudinal and transverse directions may be written by

$$\alpha_{11} = \frac{V_f E_{11}^f \alpha_{11}^f + V_m E^m \alpha^m}{V_f E_{11}^f + V_m E^m} \tag{2a}$$

$$\alpha_{22} = (1+\nu^f) V_f \alpha_{22}^f + (1+\nu^m) V_m \alpha^m - \nu_{12}\alpha_{11} \tag{2b}$$

where α_{11}^f, α_{22}^f and α^m are thermal expansion coefficients of the fiber and the matrix respectively, and the effective piezoelectric moduli e_{31} and e_{32} can be expressed by (Mallik and Ray, 2003)

$$\begin{aligned} e_{31} = V_f e_{31}^f - (V_m V_f / H)\{ & (C_{13}^f - C_{13}^m)[(V_m C_{22}^f + V_f C_{22}^m) e_{33}^f \\ & - (V_m C_{23}^f + V_f C_{23}^m) e_{31}^f] + (C_{12}^f - C_{12}^m)[(V_m C_{33}^f + V_f C_{33}^m) e_{31}^f \\ & - (V_m C_{23}^f + V_f C_{23}^m) e_{33}^f]\} \end{aligned} \tag{3a}$$

$$\begin{aligned} e_{32} = e_{31}^f + (V_m / H)\{ & C_{22}^f[(V_m C_{23}^f + V_f C_{23}^m) e_{33}^f - (V_m C_{33}^f + V_f C_{33}^m) e_{31}^f] \\ & - C_{23}^f[(V_m C_{22}^f + V_f C_{22}^m) e_{33}^f - (V_m C_{23}^f + V_f C_{23}^m) e_{31}^f]\} \end{aligned} \tag{3b}$$

in which

$$H = (V_m C_{22}^f + V_f C_{22}^m)(V_m C_{33}^f + V_f C_{33}^m) - (V_m C_{23}^f + V_f C_{23}^m)^2 \tag{3c}$$

and e_{31}^f and e_{33}^f are the piezoelectric coefficients of the fiber, and C_{ij}^f and C_{ij}^m are the elastic constants of the fiber and the matrix, respectively. The relation between C_{ij}^f $(i,j=1\text{–}6)$ and E_{11}^f, E_{22}^f, G_{12}^f, G_{13}^f and G_{23}^f can be found in Reddy (2004) and other textbooks.

It is assumed that the material property of matrix C_{ij}^m $(i,j=1\text{–}6)$ is a function of temperature, so that all effective material properties of FRCs and PFRCs are temperature dependant.

3. Governing equations for FRC laminated cylindrical shells

Consider a circular cylindrical shell with mean radius R, length L and thickness h which consists of N plies of any kind, one of which may be PFRC. The fiber reinforcement is either uniformly distributed (UD) in each ply or functionally graded (FG) in the thickness direction. The shell is exposed to elevated temperature and is subjected to a transverse dynamic load $Q(X, Y, \bar{t})$ combined with electric loads, if any. The shell is referred to a coordinate system (X, Y, Z) in which X and Y are in the axial and circumferential directions of the shell and Z is in the direction of the inward normal to the middle surface, the corresponding displacement is designated by $\overline{U}$, $\overline{V}$ and $\overline{W}$. $\overline{\Psi}_x$ and $\overline{\Psi}_y$ are the rotations of normals to the middle surface with respect to the Y- and X-axes, respectively. The origin of the coordinate system is located at the end of the shell on the middle plane. Let $\overline{F}(X, Y)$ be the stress function for the stress resultants defined by $\overline{N}_x = \overline{F}_{,YY}$, $\overline{N}_y = \overline{F}_{,XX}$ and $\overline{N}_{xy} = -\overline{F}_{,XY}$, where a comma denotes partial differentiation with respect to the corresponding coordinates.

Based on the Sanders shell theory, Reddy and Liu (1985) developed a simple higher order shear deformation shell theory. This theory assumes that the transverse shear strains are parabolically distributed across the shell thickness. The advantages of this theory over the first order shear deformation theory are that the number of independent unknowns ($\overline{U}$, $\overline{V}$, $\overline{W}$, $\overline{\Psi}_x$ and $\overline{\Psi}_y$) is the same as in the first order shear deformation theory, but no shear correction factors are required. This theory was then extended to the case of laminates with integrated piezoelectric actuator layers by Reddy (1999). Based on Reddy's higher order shear deformation theory with a von Kármán-type of kinematic nonlinearity and including thermo-piezoelectric effects, the motion equations for shear deformable hybrid laminated cylindrical shells can be derived in terms of a stress function $\overline{F}$, two rotations $\overline{\Psi}_x$ and $\overline{\Psi}_y$, and a transverse displacement $\overline{W}$. They are

$$\tilde{L}_{11}(\overline{W}) - \tilde{L}_{12}(\overline{\Psi}_x) - \tilde{L}_{13}(\overline{\Psi}_y) + \tilde{L}_{14}(\overline{F}) - \tilde{L}_{15}(\overline{N}^P) - \tilde{L}_{16}(\overline{M}^P) - \frac{1}{R}\overline{F}_{,XX}$$