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Fitting of robust reference surface based on least absolute deviations

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Abstract

Engineering surfaces comprise shape deviations namely form, waviness and roughness. For characterization of roughness, form and waviness are separated from the measured surface by establishing a reference surface that represents these deviations. This paper presents a new approach of simultaneously separating form and waviness deviations by fitting a reference surface that remains robust against the outliers such as deep grooves. A second degree polynomial and a set of sinusoidal functions are taken as basis functions to represent form and waviness respectively. A criterion of minimization of sum of absolute deviations (L_1-norm) is considered as against the commonly used least squares (L_2-norm) criterion and the reference surface obtained is found to be robust against outliers such as deep valleys in the measured surface. The superiority of the proposed fitting scheme is brought out by testing on different surfaces and comparing with the least squares method of fitting and the robust Gaussian regression filtering.

Keywords: Waviness; Robust reference surface; Fitting; Least squares; Least absolute deviations; Levenberg–Marquardt algorithm

1. Introduction

Engineering surface data is processed to remove long-wave components of form and waviness before characterization. The developments in measurement techniques and powerful computers have enabled the three-dimensional assessment of surfaces for functional characterization. The concepts involved in profile filters such as Gaussian, wavelet, envelope and motif filters are being extended for assessment of surfaces [1–4]. Gaussian profile filtering introduced by ISO [5] is the most widely used filtering technique presently. However, Gaussian filter has certain disadvantages like boundary effect, inability of form approximation and sensitivity to deep valleys in the surface [6,7]. Several modifications such as regression procedure [7,8] and use of robust functions [9] were suggested in literature to overcome the above problems of boundary effect and sensitivity to deep valleys. For removal of form, a pre-filtering is generally performed.

Fitting of reference surface can be a better alternative to filtering approach, as it does not suffer from the boundary effects of filtering methods. Also, a mathematical representation of form and waviness would be helpful in analysis of the effects of these components on functional behavior. The form component exists in measured surfaces due to reasons like misalignment of measurement direction with nominal direction of the surface, measurement made over a curved surface etc. Generally, the form component is removed from the surface by least squares (LS) fitting of a second order polynomial. Dong et al. [10] discussed various requirements of a reference datum for form removal. Based on this analysis, it has been recommended that a second degree polynomial surface would be suitable for curvature removal of any nominally uni-curved surface since this involves lesser computation time and there is an insignificant difference of residual variances when higher order polynomials are used [1].

The removal of waviness using least squares technique (L_2-norm) is somewhat difficult though that would give unique waviness. This is because the geometrical nature of waviness is generally unknown and cannot be expressed in mathematical terms. However, the assumption of proper basis functions

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Nomenclature

a	set of parameters of fitting function
$\boldsymbol{a}_0$	initial solution for the parameter set
A, B	amplitudes of cosine and sine components of the fitting function
APSD	areal power spectral density
b	coefficients of polynomial surface
C	matrix of coefficients of the fitting function
D	augmented matrix
e	error between the measured data and estimated data
J	Jacobian of fitting function
k_{max}	maximum number of iterations in LM method
K	number of sinusoidal components
L	number of parameters in fitting function
LAD	least absolute deviations
LM	Levenberg–Marquardt
LS	least squares
M, N	number of points on the surface in x- and y-directions respectively
P	probability of the data set
r	fitted reference surface
RGR	robust Gaussian regression
S_{bi}	Surface bearing index
S_{ci}	Core fluid retention index
S_{vi}	Valley fluid retention index
v	surface ordinates represented in vector form
z	measured surface

Greek symbols

α	matrix of sum of damping factor and approximate Hessian matrix
β	vector of product of Jacobian matrix and error vector
γ	groove width fraction
$\delta\mathbf{a}$	increment in parameter values
ε_1, ε_2	fraction values for specifying convergence criteria
λ_{cx}, λ_{cy}	cutoff wavelengths in x- and y-directions respectively
μ	damping factor in LM method
ρ	minimization function
ω_x, ω_y	angular frequencies in x- and y-directions respectively

for approximation of waviness is a prerequisite in least squares (LS) technique. Shunmugam and Radhakrishnan [11] have considered a sinusoidal basis function whose wavelength is obtained from the dominant component in the power spectral density of profiles. However, single sine wave may not be able to represent the waviness. O'Connor and Spedding [12] reported the use of sinusoidal functions for representing waviness of surface profiles. In their work, the initial estimates of wavelengths are obtained from Fourier spectrum analysis and the cyclic descent method was used to optimize the value of each parameter in the fitted sinusoids.

In least squares fitting technique, the boundary effect relevant in filters can be eliminated with proper selection of basis functions for approximation of form and waviness. However, the reference surfaces fitted using least squares criterion will be greatly affected by the deep valley outliers as in the case of plateau honed surfaces. To reduce the sensitivity of fitting algorithm to outlier points, a least absolute deviations (L_1-norm) criterion can be considered where the sum of absolute deviations is minimized instead of minimization of sum of squares of deviations. However, the least absolute deviations (LAD) criterion is mathematically difficult due to non-existence of derivatives at certain points [13]. To overcome this problem, Matheson [14,15] suggested a simple modification to the existing non-linear least squares algorithms so that they can be converted into LAD problems.

In the present work, a combination of sinusoidal functions is taken as the basis functions for waviness approximation. However, they form a problem of fitting non-linear functions which cannot be solved by ordinary least squares techniques. Generally, for solving non-linear minimization problems, Taylor series approximations of functions are taken and solved for changes in parameter values starting from an initial solution using Gauss–Newton method. Many times this would lead to singularity conditions [16,17] in minimization algorithm and hence convergence problems arise. To overcome this, Levenberg–Marquardt (LM) algorithm [17] is recommended by researchers which can avoid singularity conditions by using a damping factor. The present work uses LM method to minimize the sum of absolute deviations following the method presented by Matheson [14]. To remove form effectively along with waviness, a second order polynomial expression is also included with the combination of sinusoidal functions. Few examples of form removal and robustness of reference surface against deep valley surfaces are considered to illustrate the effectiveness of present fitting method using the criterion of least absolute deviations.

2. Fitting reference surfaces to engineering surfaces

2.1. Basis functions

The reference surface has to represent both form and waviness of the measured surfaces. Therefore, a second order polynomial for approximation of form and a combination of sinusoidal functions for representing the waviness ($r(x, y)$) are considered as given below,

$$r(x,y) = b_0 + b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2 + \sum_{k=1}^{K} A_k\cos(\omega_{xk}x + \omega_{yk}y) + B_k\sin(\omega_{xk}x + \omega_{yk}y)$$
$$\text{and}\quad \omega_{xk} = \frac{2\pi}{\lambda_{xk}},\quad \omega_{yk} = \frac{2\pi}{\lambda_{yk}} \qquad (1)$$

where the parameters $b_0, b_1, \ldots, b_5$ represent the coefficients of the second order polynomial surface and A_i, B_i represent the amplitudes of cosine and sine components. λ_{xk}, λ_{yk} represent the