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Fast solution of neutron transport SP_3 equation by reduced basis finite element method

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ABSTRACT

For nuclear reactor analysis and fuel depletion analysis, the neutron transport equation has to be solved many times. Fast and accurate solution of the transport problem is demanding but necessary. In the present work, the reduced basis finite element method is used to solve the generalized eigenvalue problem formulated from the simplified P3 (SP3) neutron transport equation with the cross sections treated as parameters. Numerical tests show that a speedup of around 3500 is achieved for the three-dimensional IAEA PWR benchmark problem and a speedup of around 10100 is achieved for the two-dimensional VVER-440 reactor core with the fission cross-sections, the absorption cross-sections and the scattering cross-sections treated as parameters. Construction of reduced-order parametric models would be a promising approach to build numerical nuclear reactor for high fidelity integrated simulation.

1. Introduction

Nuclear energy has been deployed for commercial usage for more than sixty years. However, many operational and safety performance-defining phenomena are not yet able to be fully modeled by a first-principles approach due to the complicated coupling physics (such as neutronics, thermal-hydraulics and thermal-mechanics) and the complex structure of nuclear reactors. The industry and the academy have been striving to develop coupled, higher-fidelity, usable modeling and simulation capabilities to predict the performance of nuclear reactors with confidence. Through double decades of efforts, high fidelity modeling capability is accomplished by taking advantage of massively parallel computers through integrating full physics and highly refined solution modules (Gaston et al., 2015; Mahadevan et al., 2014; Chauliac et al., 2011). However, due to the prohibitively high computation cost, it is yet unfeasible to run such simulations on the whole reactor core level with all the physics resolved on fine meshes, especially in the analysis which need carry out many times of calculations such as fuel depletion analysis, three dimensional core analysis with movement of control rods and the neutronics-thermohydraulics coupling analysis.

To lower the computational cost within the many-query contexts, various reduced order strategies have been developed to replace the original large-dimension model by a reduced model of substantially smaller dimension that however still accurately captures the most important features of the phenomena being modeled. Among the various reduced order methods (ROMs), the certified reduced basis finite element method (RB-FEM) (Rozza et al., 2008; Hesthaven et al., 2016; Quarteroni et al., 2016) has witnessed a spectacular effervescence in the past decade. Its high efficiency as well as the guaranteed accuracy are ensured by a decoupling of the full-order finite element scheme and the reduced order model through an offline-online procedure. When combined with a posteriori error estimation, the online stage guarantees the accuracy of the reduced order model.

In the present work, we apply the reduced basis finite element method to solve the critical problem in nuclear reactor analysis and fuel depletion analysis, i.e., the neutron transport problem. In the former case where a neutronics-thermohydraulics coupling is usually needed, the neutron transport equation has to solved iteratively for many times. In the latter case, the problem has to be solved hundreds or even thousands of times in a complete depletion analysis. Obviously, only a reduced-order model can make it acceptable to do such analysis through high-fidelity simulations. Sartori et al. (2016) has successfully applied the RB-FEM to simulate the nuclear reactor control rods movement and obtained

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a fast-running prediction of reactivity and neutron flux distribution with the geometric domain parametrized. Very recently, we applied the RB-FEM to solve the neutron diffusion problem with the cross sections treated as parameters (Zhang and Chen, 2018). Even for three dimensional configurations, both cases have achieved near real-time predictions. However, the diffusion approximation adopted by the calculations neglects the angular dependence of the momentum of neutrons and is inadequate for advanced core analysis or fuel analysis where the transport effect is more prominent (Cho, 2005). To achieve a pin-by-pin fine-mesh resolution in the next-generation core analysis, more elaborate approximations are necessary.

In the present work, we choose the simplified P3 (SP_3) equation (Brantley and Larsen, 2000) as the basic equation to ensure solution accuracy superior to the diffusion equation. Considering the local degree of freedom is doubled and the mesh is much refined in SP_3 calculations, a reduced order model is especially attractive for practical applications. The rest of this paper is organized as follows. In Section 2, the parametrized neutron transport SP_3 equation as well as the formulated generalized eigenvalue problem are briefly introduced. In Section 3, the conventional finite element formulation as well as the reduced basis finite element formulation are derived. Two benchmark problems, i.e., the three-dimensional IAEA pressurized water reactor and the two-dimensional VVER-440 reactor, are used to examine the built order reduced model in Section 4. Finally, concluding remarks and further studies will be discussed in Section 5.

2. Parametrized neutron transport sp3 equation

The multi-group approximation, i.e., the neutron spectrum is discretized as groups, is widely used in reactor analysis. For light water reactors, the two-group approximation, i.e., the group of fast neutrons with energy above 1.0 ev (Group 1) and the group of thermal neutrons with energy below 1.0 ev (Group 2), is well accepted. Only the stationary two-group SP_3 equation (Tatsumi and Yamamoto, 2003) is considered in the study. For each group of the neutrons, the SP_3 equation consists of two coupling diffusion-like equations,

$$-\frac{27}{35}D_g\nabla^2\Phi_g^2+\sum_{t,g}\Phi_g^2-\frac{2}{5}\sum_{r,g}\Phi_g^0=-\frac{2}{5}\left(\frac{\chi_g}{K_{eff}}\sum_{g'}v\sum_{f,g'}\Phi_{g'}^0+\sum_{g'\neq g}\sum_{s,g'\to g}\Phi_{g'}^0\right) \quad (1)$$

$$-D_g\nabla^2(\Phi_g^0+2\Phi_g^2)+\sum_{r,g}\Phi_g^0=\frac{\chi_g}{K_{eff}}\sum_{g'}v\sum_{f,g'}\Phi_{g'}^0+\sum_{g'\neq g}\sum_{s,g'\to g}\Phi_{g'}^0 \quad (2)$$

For group $g(g = 1,2)$: Φ_g^0 and Φ_g^2 are respectively the neutron flux of the first order and the second order moments, D_g is the diffusion coefficient, $\sum_{t,g}$ is the total cross section, $\sum_{r,g}$ is the removal cross section, $v\sum_{f,g}$ is the production cross section, χ_g is the fission spectrum, $\sum_{s,g'\to g}$ is the scattering cross section from group g to group g' and K_{eff} is the effective multiplication factor. In additional, $\sum_{r,g}=\sum_{t,g}-\Sigma_{s,g\to g'}=\Sigma_{a,g}+\sum_{g'\neq g}^{G}\Sigma_{s,g\to g'}$ where $\Sigma_{a,g}$ is the absorption cross section. The cross sections determine the nuclear reaction rates. Eqs. (1) and (2) in fact define a generalized eigenvalue problem of which the first model represents the critical state of the reactor.

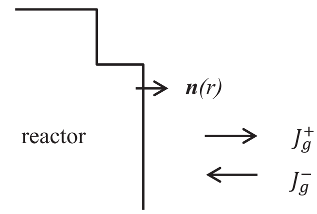

Fig. 1. Schematic of the boundary condition. $\boldsymbol{n}(r)$ denotes the outer normal direction of the boundary.

The out-going neutron current, i.e., J_g^+ and the in-coming neutron current, i.e., J_g^-, on the boundaries (Fig. 1) are defined as (Moustafa, 2016),

$$J_1^{\pm}=\frac{1}{4}(\Phi_g^0+2\Phi_g^2)\pm\frac{1}{2}(-D_g\nabla(\Phi_g^0+2\Phi_g^2))-\frac{3}{16}\Phi_g^2 \quad (3)$$

$$J_3^{\pm}=\frac{3}{80}(\Phi_g^0+2\Phi_g^2)\pm\frac{1}{2}\left(-\frac{27}{35}D_g\nabla\Phi_g^2\right)-\frac{21}{80}\Phi_g^2 \quad (4)$$

In the present study, the boundary is assumed to be vacuum (i.e., $J_1^-=0$ and $J_3^-=0$) and it be represented as a Robin boundary condition. A similar treatment also applies to other types of boundary conditions.

3. Finite element formulation of the generalized eigenvalue problem

3.1. Finite element formulation of the generalized eigenvalue problem

With definition $\hat{\boldsymbol{\Phi}}_g^0\equiv\boldsymbol{\Phi}_g^0+2\boldsymbol{\Phi}_g^2$, the weak forms of the two coupling equations can be written as,

$$\frac{2}{5}\sum_{r,g}\int_\Omega\hat{\Phi}_g^0 v d\Omega-\frac{27}{35}D_g\int_\Omega\nabla\Phi_g^2.\nabla v d\Omega-\left(\sum_{t,g}+\frac{4}{5}\sum_{r,g}\right)\int_\Omega\Phi_g^2 v d\Omega$$
$$=\frac{2}{5}\Big(\sum_{g'\neq g}^{G}\sum_{sg'\to g}\int_\Omega\Phi_{g'}^0 v d\Omega+\frac{\chi_g}{K_{eff}}\int_\Omega\sum_{g'}v\sum_{f,g'}\Phi_{g'}^0 v d\Omega\Big)$$
$$+\frac{27}{35}D_g\int_{\partial\Omega}\frac{\partial\Phi_g^2}{\partial n}\cdot v d\partial\Omega \quad (5)$$

$$D_g\int_\Omega\nabla\hat{\Phi}_g^0\cdot\nabla v d\Omega+\sum_{r,g}\int_\Omega\hat{\Phi}_g^0 v d\Omega-2\sum_{r,g}\int_\Omega\Phi_g^2 v d\Omega$$
$$=\sum_{g'\neq g}^{G}\int_\Omega\sum_{sg'\to g}\Phi_{g'}^0 v d\Omega+\frac{\chi_g}{K_{eff}}\int_\Omega\sum_{g'}v\sum_{f,g'}\Phi_{g'}^0 v d\Omega$$
$$+D_g\int_{\partial\Omega}\frac{\partial\hat{\Phi}_g^0}{\partial n}\cdot v d\partial\Omega \quad (6)$$

where Ω and $\partial\Omega$ represent the domain of the problem and the corresponding boundary, respectively.$v\in V$ is the test function and V is a Hilbert space with an induced norm $\|\cdot\|_V=\sqrt{(\cdot,\cdot)_V}$. To see the affine-dependence clearly, the parameters, i.e., the cross sections, are written separately ahead of the corresponding integral components.

The approximate solution of the neutron flux can be expressed as the combination of the basis functions $\{\varphi_i, i=1,2,\ldots,N_v\}$, i.e., $\hat{\Phi}_g^0\approx\sum_{i=1}^{N_v}\tilde{\Phi}_{g,i}^0\cdot\varphi_i$ and $\Phi_g^2\approx\sum_{i=1}^{N_v}\tilde{\Phi}_{g,2}^2\cdot\varphi_i$ where $\tilde{\Phi}_{g,0}^i$ and $\tilde{\Phi}_{g,2}^i$