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Probability distribution for neutron density of two energy group point kinetics system

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ABSTRACT

The probability distribution of the number of neutrons and delayed neutron precursors in a multiplying assembly of various types of reactivities is developed for two energy groups. The space independent point reactor kinetics model for six precursors group of delayed neutrons is considered. The problem is formulated in terms of the probability distribution, generating function which satisfies a partial differential equation, derived in this paper. The probability distribution of two energy group delayed neutrons and the density of the precursors are obtained by solving this system of reactor kinetics model by adopting the mathematical methods. At the point, when the system is formulated in an operator or matrix form, the Magnus expansion furnishes an elegant setting to build up approximate exponential representations of the solution of the kinetics system. It provides a power series expansion for the corresponding exponent and is sometimes referred to as time-dependent exponential perturbation theory. Every Magnus approximate corresponds in perturbation theory to a partial re-summation of infinite terms with the important additional property of preserving, in any order, certain symmetries of the exact solution. The first, second and third Magnus expansions are described and used to predict the first moment of fast, thermal and multi-group of delayed neutrons precursor for the two-energy point kinetics reactor system. The validity of the presented method is tested with the aid of the eigenvectors and eigenvalues of the kinetics system in the matrix form by comparing with the conventional methods.

1. Introduction

The mathematical systems of reactor kinetics are normally represented in terms of a collection of individuals governed by the competition of the two basic random mechanisms of birth and death. The neutron population in a nuclear reactor is subject to fluctuations in time and in space due to the competition of diffusion by scattering, births by fission events, and deaths by absorptions. The common textbook introduction to the neutron transport equation or its successive approximations is based on a particle balance argument for the "average number of particles" within an element of phase space. The result is a linear, deterministic equation for the angular flux $\Psi(r,v,t) = vn(r,v,t)$, e.g., (Duderstadt et al., 1976; Hetrick, 1971; Stacey, 2007; Glasstone et al., 1981; Ray, 2016; Lamarsh and Baratta, 2001). Although the neutron transport equation does not include any description of stochasticity, the quantity $n(r,v,t)$ is interpreted as the 'mean' particle density.

Stochastic models of nuclear reactors have been employed by several authors (e.g. (Courant and Wallace, 1947; Feynman, 1946; Feynman et al., 1956; Hoffmann, 1949; Hansen, 1960; Gillespie, 1992)). The new derivation of the two energy group point kinetics system presented in this paper allow to have the moments of the probability distribution, where all important information for the system's behavior can be had through this moments. The first moment relates to the mean of the probability distribution, the second to the variance, the third to skewness and the fourth to kurtosis. The predicted values of different type of neutrons play a major rule for computing the power reactor and in a study the safety of nuclear reactors. For this aim, we derive a system of stiff two-energy-group point kinetics differential equations from the perspective of probability theory. In most cases, attention was confined to the first two moments of the probability distribution, where the behavior of the first moment was given by the usual reactor kinetic equations and the behavior of the second moment was determined from the stochastic model. More insight is gained with the statistical mechanics approach in which the neutron transport equation is seen as a linear form of Boltzmann's equation

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that describes the approach towards equilibrium of a weakly dense gas. The qualitative description of this picture is that of particles moving freely along deterministic trajectories and undergoing punctual interactions; the random behavior is thus introduced through the mechanism of collisions. In Boltzmann's picture we are able to formally identify the average number of particles with the one particle distribution function, that is, with the average number of particles at a location in phase space, regardless of the distribution of particles elsewhere (Sanchez, 1997). It is clear that, in order to have good statistics, one has to have lots of particles and lots of target nuclei. If the number of particles is small, then one suspects that a description of only the average value will not be sufficient. As the number of particles increases, we expect the statistics to improve and get into the onset of determinism.

Deriving the stochastic equations for the probability distribution as a function of time in a space-independent (point) multiplying assembly with time-independent, velocity-dependent cross sections is illustrated by Sanchez (1997). The probability distribution of the number of neutrons and delayed neutron precursors in a multiplying assembly is considered by Bell (1963). Particular emphasis is placed on the probability distribution for a system which is brought, to a supercritical state in the presence of a neutron source which is so weak that deviations from the average population may be large. A space independent model is used with one group of neutrons. The analysis has been carried on in the presence of sources, (Hansen, 1960), while accounting for precursors, (Bell, 1963). Numerical techniques based on the characteristic methods have also been applied to the evaluation of time-dependent probabilities characterizing a fission chain, (Bell et al., 1963), in the presence of precursors. The general analysis has been extended to the study of finite systems (Bell, 1965; Degweker, 1994).

In the present paper, a two-group point reactor kinetics model of fast, thermal and six groups precursors of delayed neutrons is presented. The probability balance equation with a general source term is written in terms of probability generating function. The algorithm is derived by means of analytical and statistical mathematical tools. The Magnus expansion, (Magnus, 1954; Blanes et al., 2009), is described and used to predict the first moment of fast, thermal and precursors concentration of group i for the multi-energy point kinetics reactor system. As a part of the bibliography history in the field of two energy groups for reactor kinetics, Blanchon simulate the neutron kinetics with two energy groups and six precursor groups by numerical method (Blanchon et al., 1988). The iterative algorithm and the stability has been evaluated for propose of the solution of a large linear system. The diffusion equation of neutrons in slab geometry is solved by Lemos et al. (2008) for a model with two energy groups by the technique of Laplace transform. PWS code has been developed to include a numerical solution for the time-dependent neutron diffusion equations for the nuclear reactor analysis. The new technique employs a new parameter α which can reduce the rapid increase in magnitude of the power series coefficients. The validity of the algorithm was tested with three kinds of well-known two-energy group benchmark problems Abonaber and Hamada (2009). The problem in Cartesian geometry was solved successfully by Ceolin et al. (2011) and was extended for different geometry. The analytical solution for the two-group kinetics neutron diffusion equations is introduced by Fernandes et al. (2011) in cylindrical geometry by the Hankel transform. Fernandes et al. (2013) discussed the kinetics neutron diffusion equation in homogeneous cylinder geometry. They construct solutions unaffected by a numerical artifact, known as the stiffness of the equation system, for two energy groups, one and six precursor concentrations, respectively (Fernandes et al., 2013). A novel analytical formulation is constructed and converged to high accuracy from the merger of the piecewise constant functions over a partition in time into the fundamental matrix for the two-energy group of the point kinetics equations by Aboanber et al. (2014). The resulting system of stiff linear and/or nonlinear differential equations for an arbitrary number of delayed neutrons is solved exactly over each time step.

The paper is organized as follows: the fundamental partial differential equation for the probability generating function, which representing the probability density of the neutron flux for fast and thermal energy groups and the contributions from the precursor, is derived for the two energy group model in Section 2. This basic system was converted to the matrix differential equation, which is solved based on the Magnus expansion for step, ramp and sinusoidal reactivity variations in Sections 3.

2. Basic mathematical model

The neutron in the multiplying system will disappear from the system either by leakage or by suffering a non-elastic collision (such as fission, radiative absorption, etc.). The derivation of an analytical formulation of the system of differential equations representing the probability density of the neutron flux for fast and thermal energy groups and the contributions from the precursors is the main focus of this section. Let us consider the following kinetic parameters, v_1 and v_2 are the fast and thermal neutrons speed, D_1 and D_2 are the fast and thermal diffusion coefficients, Σ_{a_1} and Σ_{a_2} are the fast and thermal absorption cross sections, Σ_{f_1} and Σ_{f_2} are the fast and thermal fission cross sections, $\Sigma_{s_{12}}$ is the scattering cross section from fast to thermal neutron, Σ_{c_1} and Σ_{c_2} are the fast and thermal capture cross section, Σ_{l_1} and Σ_{l_2} are the fast and thermal leakage cross sections, λ_i is the decay constant of $i-$group of delayed neutrons and β_i is the fraction of $i-$group delayed neutrons.

In the multiplying system, let us define l_1 and l_2 as the fast and thermal neutrons lifetime. Then the probability of fast and thermal neutrons disappearing (either by leakage or by fission and radiative absorbtion) in time interval dt are given by

$$\left.\begin{array}{ll} \frac{dt}{l_1} = v_1\Sigma_{s_{12}}dt + v_1\Sigma_{a_1}dt + v_1\Sigma_{l_1}dt, & \Sigma_{l_1} = D_1B^2 \\ \frac{dt}{l_2} = v_2\Sigma_{a_2}dt + v_2\Sigma_{l_2}dt, & \Sigma_{l_2} = D_2B^2 \end{array}\right\} \quad (1)$$

where B^2 is the material buckling in time interval dt, $v_1\Sigma_{l_1}dt$ and $v_2\Sigma_{l_2}dt$ are the probability of fast and thermal neutrons disappearing by leakage, $v_1\Sigma_{a_1}dt$ and $v_2\Sigma_{a_2}dt$ are the probability of fast and thermal neutrons disappearing by absorption and $v_1\Sigma_{s_{12}}dt$ is the probability of fast neutron disappearing by scattering from fast to thermal. Precursors of type i are characterized by lifetime $\frac{1}{\lambda_i}$ such that a precursor has probability $\lambda_i dt$ of producing a neutron in time interval dt. The neutron source is characterized by $S_k(t)$, $k = 1, 2, 3, \cdots, K$, where $S_k(t)dt$ is the probability that the neutron source emitted k neutrons during time interval dt.

Let us consider the probability that at time t there are exactly n_1 fast neutrons, n_2 thermal neutrons and m_i precursors of type i in the system $(i = 1, 2, \cdots, I)$

$$P(n_1, n_2, m_1, \cdots, m_I, t) = P(n_1, n_2, \mathbf{m}, t)$$

Here $\mathbf{m}$ is a vector with components $m_1, m_2, \cdots, m_I$. By enumerating all the possible events that can happen in the interval dt such as:

$\left[1 - \frac{n_1 dt}{l_1} - \frac{n_2 dt}{l_2} - \sum_{i=1}^{I}\lambda_i m_i dt - S_k(t)dt\right]P(n_1, n_2, \mathbf{m}, t)$ is the probability that there is no change in the number of fast, thermal neutrons and precursors of type i in time dt, $v_1\Sigma_{s_{12}}(n_1+1)dtP(n_1+1, n_2-1, \mathbf{m}, t)$ is the probability of missing fast neutron due to scattering from fast to thermal, $v_1(\Sigma_{c_1} + \Sigma_{l_1})(n_1+1)dtP(n_1+1, n_2, \mathbf{m}, t)$ and $v_2(\Sigma_{c_2} + \Sigma_{l_2})(n_2+1)dtP(n_1, n_2+1, \mathbf{m}, t)$ are the probability that fast and thermal neutrons disappear from the system either by capture or by leakage respectively,