Annals of Nuclear Energy 119 (2018) 322–330

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Reactor core power mapping based on Bayesian inference

Xingjie Peng *, Dong Qing, Yun Cai, Rui Guo, Wenhao Ji, Shuai Wang, Yingrui Yu, Qing Li

Science and Technology on Reactor System Design Technology Laboratory, Nuclear Power Institute of China, Chengdu, China

ARTICLE INFO

Article history:
Received 21 February 2018
Received in revised form 3 May 2018
Accepted 5 May 2018
Available online 26 May 2018

Keywords:
Power mapping
Bayesian inference
Kalman filter
Coupling coefficients

ABSTRACT

Bayesian inference provides a coherent probabilistic approach for combining information from measurements of in-core neutron detectors and numerical neutronics simulation results, and thus is an appropriate framework for reactor core power mapping which has been implemented in this paper. Measurements from DayaBay Unit 1 PWR are used to verify the accuracy of the Bayesian inference method, and comparisons are made among the Bayesian inference method, the Kalman filter method and the very-often-used coupling coefficients method. The root mean square errors (RMSE), the maximum relative errors (MRE), and the power peak reconstruction error (PPRE) of the Bayesian inference method are less than 0.31%, 1.64% and 0.07% for the entire operating cycle separately. The reconstructed assembly power distribution results and the calculation speed show that the Bayesian inference method is a promising candidate for on-line core power distribution monitoring system.

1. Introduction

Reactor core power mapping, i.e. power distribution monitoring is vital to core surveillance of operating power reactors, and detailed three dimensional power distribution serve as one of the basic operation parameters which can directly determine many other important parameters such as power peaking factor, enthalpy rising factor and quadrant tilt ratio to evaluate the safety margins and optimize the economy of nuclear reactors. Most commercial power reactors in operation are equipped with movable or fixed in-core neutron detectors to obtain useful power distribution information, and sustained efforts have been devoting to develop on-line monitoring systems, such as BEACON (Boyd and Miller, 1996), using fixed in-core detectors for core surveillance of the third-generation nuclear power plants. The signals of detector at certain location reflect the actual reactor flux or power can be applied to improve the results of the only theoretical diffusion calculations.

The path to develop fast and accurate power mapping method has never been stopped due to the increasing needs of ensuring safety and optimizing economy of nuclear power plants. A process of least-squares fitting of the measured vanadium detector signals to a linear expansion of pre-calculated flux modes is implemented in the CANDU on-line flux mapping system (Tang et al., 1978) to map out 3D flux distribution. Coupling coefficients method (Karlson, 1995; Jang, 2004) determines the powers of the uninstrumented assemblies through the solving of a linear system containing coupling coefficients, and several improved methods such as Lagrange multiplier method (Webb and Brittingham, 2000) have been developed. Lee and Kim (Lee and Kim, 2003) proposed a least-squares method by combining the coarse mesh finite difference (CMFD) form of the fixed-source diffusion equation and the detector response equation to form an over-determined linear equation. Peng (Peng, 2014) utilized the ordinary kriging method to perform power mapping and optimize the locations of fixed detectors. Li (Li, 2014) proposed an on-line monitoring method based on the nodal method and the harmonics synthesis method. Data assimilation techniques, such as three dimensional variational optimization method (3D-VAR) (Massart and Buis, 2007) and its variant Kalman filter (Bertrand, 2012), have also been applied in neutronic field interpolation to estimate power distribution in some optimal way using measurements and computer simulations.

Bayesian inference (Bishop, et al., 2006) is defined as the process of fitting a probability model to a set of data and summarizing the result by a probability distribution on the parameters of the model and on unobserved quantities such as predictions for new observations. In nuclear engineering field, Bayesian inference has been applied in inverse uncertainty quantification to get the probability density function of model parameters that are consistent with the experimental data. Wu (Wu, 2018) performed the inverse uncertainty quantification of the model parameters of BISON fission gas release (FGR) model based on Risø-AN3 benchmark FGR time series data. Li (Li, 2017) assessed the uncertainty for model parameters of RELAP5 code related to reflood phenomena

* Corresponding author.
E-mail addresses: pengxingjiets@gmail.com, pengxingjiets@126.com (X. Peng).

https://doi.org/10.1016/j.anucene.2018.05.014
0306-4549/

with data of FEBA (Flooding Experiments with Blocked Arrays) facility. Castro (Castro, 2016) proposed a Bayesian inference model MOCABA to utilize measurement information obtained in the previous cycle to reduce the prediction uncertainties of the boron letdown curve and the fuel assembly-wise power distribution of the current cycle. In this paper, we introduce Bayesian inference into reactor core power distribution mapping to reconstruct the whole core power distribution based on neutronics simulation and measurements of in-core neutron detectors, and compare its results with those of the Kalman filtering method which was proposed in reference (Bertrand, 2012).

2. Methodology

2.1. Bayesian inference

Let us consider an arbitrary vector function $\mathbf{x}$, and $\mathbf{x}$ is a random vector defined by a prior probability density function (pdf) $p(\mathbf{x})$ which reflects the uncertainty of $\mathbf{x}$ without any knowledge of measurements and is generally assumed to be a multivariate normal distribution characterized by a mean vector $\mathbf{x}_0$ and a covariance matrix Σ_0.

According to Bayes' theorem, the updated information about $\mathbf{x}$ is characterized by the posterior pdf which is defined as:

$$p(\mathbf{x}|\mathbf{v}) = \frac{p(\mathbf{v}|\mathbf{x})p(\mathbf{x})}{\int p(\mathbf{v}|\mathbf{x})p(\mathbf{x})d\mathbf{x}} \propto p(\mathbf{v}|\mathbf{x})p(\mathbf{x}) \quad (1)$$

where $\mathbf{v}$ represents the vector defining the measurements, $p(\mathbf{v}|\mathbf{x})$ is the likelihood function which can also be represented by a normal distribution, and then a multivariate normal posterior pdf $p(\mathbf{x}|\mathbf{v})$ characterized by mean vector $\mathbf{x}^*$ and a covariance matrix Σ^* can be generated.

In this work, we applied the description of Bayesian inference adopted by Castro and partition the model parameters of the prior distribution, the likelihood function and the posterior distribution into an application case A and a benchmark case B in order to distinguish them:

$$\begin{aligned}
&\mathbf{x}_0 = (\mathbf{x}_{0A}; \mathbf{x}_{0B}), \quad \Sigma_0 = \begin{pmatrix} \Sigma_{0A} & \Sigma_{0AB} \\ \Sigma_{0AB}^T & \Sigma_{0B} \end{pmatrix}, \\
&\mathbf{v} = (\mathbf{v}_A; \mathbf{v}_B), \quad \Sigma_V = \begin{pmatrix} \Sigma_{VA} & \mathbf{0} \\ \mathbf{0}^T & \Sigma_{VB} \end{pmatrix}, \\
&\mathbf{U} = \begin{pmatrix} \mathbf{U}_A & \mathbf{0} \\ \mathbf{0}^T & \mathbf{U}_B \end{pmatrix}, \quad \mathbf{x}^* = (\mathbf{x}_A^*; \mathbf{x}_B^*), \\
&\Sigma^* = \begin{pmatrix} \Sigma_A^* & \Sigma_{AB}^* \\ \Sigma_{AB}^{*T} & \Sigma_B^* \end{pmatrix}.
\end{aligned} \quad (2)$$

where Σ_V represents the measurement covariance and $\mathbf{U}$ represents the measurement operator which describes the mapping relationship between $\mathbf{x}$ and $\mathbf{v}$.

In order to obtain $\mathbf{x}^*$ and Σ^*, the posterior pdf $p(\mathbf{x}|\mathbf{v})$ has to be maximized with respect to $\mathbf{x}$. Under the assumption that we only have direct measurements of $\mathbf{x}_B$, then the following expressions for the posterior model parameters can be obtained:

$$\mathbf{x}_A^* = \mathbf{x}_{0A} + \Sigma_{0AB}(\Sigma_{0B} + \Sigma_{VB})^{-1}(\mathbf{v}_B - \mathbf{x}_{0B}) \quad (3)$$

$$\mathbf{x}_B^* = \mathbf{x}_{0B} + \Sigma_{0B}(\Sigma_{0B} + \Sigma_{VB})^{-1}(\mathbf{v}_B - \mathbf{x}_{0B}) \quad (4)$$

$$\Sigma_A^* = \Sigma_{0A} - \Sigma_{0AB}(\Sigma_{0B} + \Sigma_{VB})^{-1}\Sigma_{0AB}^T \quad (5)$$

$$\Sigma_B^* = \Sigma_{0B} - \Sigma_{0B}(\Sigma_{0B} + \Sigma_{VB})^{-1}\Sigma_{0B} \quad (6)$$

$$\Sigma_{AB}^* = \Sigma_{0AB} - \Sigma_{0AB}(\Sigma_{0B} + \Sigma_{VB})^{-1}\Sigma_{0B} \quad (7)$$

We can see from Eqs. (3)–(5) that the understanding of $\mathbf{x}_A$ can be updated with the measurement $\mathbf{v}_B$ and its corresponding covariance Σ_{VB}.

2.2. Application in power distribution mapping

Bayesian inference is applied only to the 2-dimensional radial power distribution mapping in this study, and the axial power distribution can be reconstructed by the cubic spline synthesis method (Wang, 1991) in the real application of core power distribution monitoring system. In the following contents, we will see that Bayesian inference isn't used directly in nodal power estimation, but in calibration factor estimation.

In the real application of core monitoring system, the detector current signals are transformed into measured nodal powers of instrumented assemblies by using the power-to-signal ratio which can be determined from fine-mesh, multigroup diffusion theory (Webb and Brittingham, 2000). After obtaining the measured nodal powers, the model calibration factor of instrumented assembly is defined by

$$\Delta_i = P_i^{\text{mea}} / P_i^{\text{cal}} \quad (8)$$

where i labels the location of the instrumented assembly, P_i^{mea} is the corresponding measured nodal power, and P_i^{cal} is the corresponding predicted nodal power calculated by neutronics code.

Bayesian inference is then applied to get the calibration factors of the un-instrumented assemblies given those factors of instrumented assemblies by the utilization of Eq. (3) which can be rewritten into:

$$\Delta_j^* = \Delta_{0j} + \Sigma_{0ji}(\Sigma_{0i} + \Sigma_{Vi})^{-1}(\Delta_i - \Delta_{0i}) \quad (9)$$

where j labels the location of the un-instrumented assembly, Δ_j^* represents the calibration factors of the un-instrumented assemblies, $\Delta_{0i/j}$ represent the mean values of the prior distribution of calibration factors for all assemblies, and the measurement vector is Δ_i. Finally, the reconstructed nodal power of un-instrumented assembly can be obtained:

$$P_j^{\text{recon}} = \Delta_j^* P_j^{\text{cal}} \quad (10)$$

where P_j^{recon} is the reconstructed power distribution of the un-instrumented assemblies, and P_j^{cal} is the predicted nodal power calculated by neutronics code.

In the application of Bayesian inference, the mean values of calibration factors of the prior distribution $\Delta_{0i/j}$ are set as one:

$$\Delta_0 = (\Delta_{0j}; \Delta_{0i}) = \mathbf{I} \quad (11)$$

In this study, the second order auto-regressive function (Massart and Buis, 2007) is used to specify the covariance matrices Σ_{0ji}, Σ_{0j} and Σ_{0i} to reflect prior information. In such a function, the amount of covariance depends from the Euclidean distance between spatial points. The correlation length L has different values, which means we are dealing with a global pseudo Euclidean distance. The element of all covariance matrices that shows the correlations between calibration factors of two assemblies can be expressed as:

$$\Sigma_{0mn} = \sigma^2\left(1 + \frac{r_{mn}}{L}\right)\exp\left(-\frac{r_{mn}}{L}\right) \quad (12)$$

where r_{mn} represents the spatial distance between two assemblies labeled by m and n, σ represents the standard deviation. There is an assumption that all the random variables have the same value of σ.

The measurement covariance Σ_{Vi} is approximated by a diagonal matrix and this means that no direct correlation exists between