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Neutron transport solution of lattice Boltzmann method and streaming-based block-structured adaptive mesh refinement

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ABSTRACT

Simulation of neutron transport problem is the kernel of nuclear reactor physics, whose application, however, is limited by the exorbitant computational cost and complex geometry structure. This paper presents a lattice Boltzmann method (LBM) for multi-group neutron transport process and proposes a streaming-based block-structured adaptive-mesh-refinement (SSAMR) technique. The neutron lattice Boltzmann equation is deduced from the neutron transport equation and the macroscopic neutron diffusion equation can be recovered from neutron lattice Boltzmann equation via the Chapman-Enskog expansion, which makes the kinetic significance of lattice Boltzmann equation clearly. The significance of relaxation time for neutron LBM is further discussed for the first time, and the factors affecting the neutron relaxation process are studied deeply also. After establishing the neutron LBM, the SSAMR technique is applied to efficiently utilizing the computational resources of proposed LBM. To simply achieve the data communication between different meshes and eliminate the discontinuity of scalar neutron flux, a data exchange technique based on the streaming process of LBM is adopted. Simulation results show that the proposed LBM can be applied to solving neutron transport process in all dimensions, and the SSAMR technique can not only effectively reduce the computational cost, but also be easily implemented. This work may provide some new perspectives for solving the neutron transport process and a powerful thought for large and complex engineering calculation.

1. Introduction

Propagation of neutron particles within a scattering and absorbing medium has received a substantial amount of interests from within the research community, including astrophysics, nuclear reactor physics, nuclear reactor design and neutron capture therapies (mainly in boron neutron capture therapy) (Barth, 2003; Oka, 2014). Theoretically, this process can be described by neutron transport equation (NTE) (Davison, 1957), and its integro-difference nature makes this highly dimensional equation difficult to solve. In recent years, researchers have been devoting to the numerical solution of neutron transport problem, including numerical technique and computational efficiency.

Over the last half century, a vast array of numerical techniques have been established and developed on both statistical approach, i.e., the Monte Carlo method (MCM) (Guo et al., 2017; Li et al., 2016a, 2018; Wang et al., 2015), and the deterministic technique, such as the discrete-ordinate method (DOM, also be known as S_N) (Owens et al., 2016), the spherical harmonics method (P_N) (Brunner and Holloway, 2005), the method of characteristic (MOC) (Liu et al., 2017; Liu et al., 2011; Zhang et al., 2011) and the finite element method (FEM) (Mercimek and Özgener, 2014). The MCM is valuable for its high precision and strongly capability of treating complex geometry (Modest, 2003). Whilst all these techniques have been applied to varying degrees of success, the deterministic methods, perhaps, have received more attention owing to its higher efficiency. The main idea of the deterministic method is to simulate the neutron transport processes by reducing and solving a series of mathematical physical equations. In solving the NTE, one can adopt the S_N or P_N method to reduce the angular dependence and solves it by some mature numerical techniques such as MOC and FEM. However, these techniques require an exorbitant computational cost for solving the multi-dimensional problems. In addition, for the multi-physical nature in nuclear reactor, the coupling for neutronics and thermal-hydraulics should be considered for better safety analysis of nuclear reactor (Bindra and Patil, 2012). Thus, an algorithm suitable for coupling solution and with lower computational cost should be developed.

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In comparison, the lattice Boltzmann method (LBM) may be an optional deterministic method, which solves a series of simple linear lattice Boltzmann equations (LBE) instead of the original governing equation (Peng et al., 2006). In solving the hydrodynamic, the LBM shows some excellent characters, such as simple construction, natural mesoscopic foundation, high efficiency of parallel computations and easy treatment for complex boundaries (Guo and Shu, 2013; Succi, 2001). Due to its attractive advantages, the LBM has been extended to many other fields, such as partial difference equations (Chai et al., 2008), multiphase flow (Li et al., 2016b; Wang et al., 2016), inviscid compressible flow (Li et al., 2009), porous medium (Liu and He, 2016; Zhao et al., 2016), phonon transport (Guo and Wang, 2016), and others (Bernaschi et al., 2001; Liu and He, 2015; Shan and Chen, 1993; Xuan et al., 2007; Yan et al., 2013). More recently, some researchers extended the LBM to the particle transport problem and received some good solutions. Mishra et al. (Asinari et al., 2010; Mishra et al., 2014; Mishra and Vernekar, 2012; Vernekar and Mishra, 2014) developed the LBM for steady-state radiative transfer problem and obtained good results. Bindra et al. (Bindra and Patil, 2012; Gairola and Bindra, 2017) extended the LBM for steady-state neutron transport in source-driven condition and studied the coupled problems on the basis of LBM (Mcculloch and Bindra, 2016), indicating that the strongly coupling capability of LBM makes it possible for easily achieving full-coupled multi-physics calculation. To establish a physics-based framework rather than a partial differential equation solver, the LBE should be derived based from Chapman-Enskog expansion and the corresponding physical meaning of each parameter should be found out.

Although a good numerical method can effectively improve the computational efficiency, it cannot be ignored that the computational accuracy is closely related to the computational cost. The larger computational capacity is required by the higher resolution. With the developments of computational speed and memory, a larger scale computation has become more feasible, but it is still insufficient for full region fine calculation by improving the hardware alone. On the other hand, in nuclear reactor simulation with large variations in the spatial dimension, finer resolution is only required near the assembly boundaries due to the mutation of the media properties such as macroscopic scattering and absorbing cross sections, whilst other domains can be solved with coarser mesh. For these reasons, the adaptive-mesh-refinement (AMR) technique (Donat et al., 2014; Lovett et al., 2015; Tölke et al., 2006) is an optional technique, which adopts non-uniform grids to efficiently reduce the computational cost in the case of guaranteed accuracy. According to this technique, the finer mesh is only adopted in the positions where higher spatial resolution is required, then the computational resources can be efficiently saved with reasonable accuracy solutions (Gourma et al., 2013). This advantage makes the AMR technique been widely adopted in numerical simulations, including partial difference equations (Tang et al., 2003), shock-induced combustion (Yuan and Tang, 2007), and multi-pahse flow (Hu et al., 2009). Recently, some researchers have studied the AMR technique for neutral particle transport with DOM (Baker, 2002; Lathouwers, 2011), FEM (Mirza et al., 2007; Ragusa and Wang, 2010; Wang, 2009) and spectral element method (Nahavandi et al., 2015). Moreover, the block-structured adaptive-mesh-refinement (SAMR) technique (Baker, 2002; Fakhari and Lee, 2014; Hittinger and Banks, 2013; Luitjens and Berzins, 2011; Ralf, 2011) is a branch to simplify the implementation of AMR technique, which covers the computational domain with a series of blocks, and applies the structured uniform mesh to each block. Comparing to the AMR technique, the SAMR technique eliminates the difficult tree traversal for finding the nearest neighbors of a given cell, and the tree-type data structure (Teyssier, 2002) is actually removed. In addition, the computational memory requirement for storing the data structure is also reduced (Fakhari and Lee, 2014). In contrast to the previous works, the LBM could be more convenient to incorporate with the SAMR technique due to its simple implementation to solve the neutron transport problem.

This paper focuses on two open issues on the neutron transport modeling. The first issue concerns on the approach of establishing the LBM for solving the neutron transport problem, determining the physical meaning of the neutron lattice Boltzmann equation (NLBE) and the unknown parameters. The second one concerns on implementing the SAMR technique for the proposed LBM, and reducing the amount of computational resources.

The remainder of this article is organized as below. In Section 2, the detailed derivation of the neutron LBM is introduced, including the neutron transport equation, the neutron LBM, the recovery of macroscopic diffusion equation and the treatments of boundary conditions. In Section 3, a streaming-based block-structured adaptive-mesh-refinement (SSAMR) technique for present LBM is proposed in detailed and the implementation is listed at the rest of this section. Section 4 tests three numerical benchmark problems to verify the LBM and the SSAMR technique. The concluding remarks are finally summarized in Section 5.

2. Lattice Boltzmann equation for neutron transfer

In this section, the NTE is converted to a NLBE and the macroscopic diffusion theory are recovered from the NLBE via the Chapman-Enskog expansion. The treatments of typically vacuum, bare and reflective boundary conditions are listed subsequently.

2.1. Neutron lattice Boltzmann model

Theoretically, the process of neutral particle propagation in a homogeneous scattering and absorbing medium can be governed by a linear BTE, and the NTE is its branch (Duderstadt and Hamilton, 1976)

$$\frac{1}{v}\frac{\partial\psi(\boldsymbol{r},\Omega,t)}{\partial t}+\Omega\cdot\nabla\psi(\boldsymbol{r},\Omega,t)+\Sigma_t(\boldsymbol{r},t)\psi(\boldsymbol{r},\Omega,t)=\int_{4\pi}\Sigma_s(\boldsymbol{r},t)f(\boldsymbol{r},\Omega'\to\Omega)\psi(\boldsymbol{r},\Omega',t)\mathrm{d}\Omega'+Q(\boldsymbol{r},\Omega,t) \tag{1}$$

where v represents the speed of a neutron particle transporting within the medium (cm/s); $\boldsymbol{\Omega}=\boldsymbol{i}\mu+\boldsymbol{j}\eta+\boldsymbol{k}\xi=\boldsymbol{i}\sin\theta\cos\varphi+\boldsymbol{j}\sin\theta\sin\varphi+\boldsymbol{k}\cos\theta$ is the angular direction with θ and φ being zenith and circumferential angle, respectively; $\psi(\boldsymbol{r},\boldsymbol{\Omega},t)$ is the angular neutron flux distribution corresponding to the position $\boldsymbol{r}$, angular direction $\boldsymbol{\Omega}$ and time t; $\Sigma_t(\boldsymbol{r},t)$ is the macroscopic total cross section and $\Sigma_s(\boldsymbol{r},t)$ is the macroscopic scattering cross section (cm^{-1}); $\Sigma_a(\boldsymbol{r},t)$ is macroscopic absorbing cross section which satisfies $\Sigma_a(\boldsymbol{r},t)=\Sigma_t(\boldsymbol{r},t)-\Sigma_s(\boldsymbol{r},t)$; $Q(\boldsymbol{r},\boldsymbol{\Omega},t)$ represents the angular source term; $f(\boldsymbol{r},\boldsymbol{\Omega}'\to\boldsymbol{\Omega})$ is the scattering phase function from direction $\boldsymbol{\Omega}'$ to direction $\boldsymbol{\Omega}$. For an isotropic scattering condition, $f(\boldsymbol{r},\boldsymbol{\Omega}'\to\boldsymbol{\Omega})=1/4\pi$.

Multiplying the velocity v simultaneously on both sides of the Eq. (1), one obtains

$$\frac{\partial\psi(\boldsymbol{r},\Omega,t)}{\partial t}+\boldsymbol{v}\cdot\nabla\psi(\boldsymbol{r},\Omega,t)=v\left(\int_{4\pi}\Sigma_s(\boldsymbol{r},t)f(\boldsymbol{r},\Omega'\to\Omega)\psi(\boldsymbol{r},\Omega',t)\mathrm{d}\Omega'-\Sigma_t(\boldsymbol{r},t)\psi(\boldsymbol{r},\Omega,t)+Q(\boldsymbol{r},\Omega,t)\right) \tag{2}$$

where $\boldsymbol{v}=v\boldsymbol{\Omega}$ is the particle's velocity vector.

In light of discrete ordinate, the continuous angle domain can be divided into a series of non-overlapping intervals. Integrating the Eq. (2) over each region, one can obtain the discrete-ordinate NTE