Annals of Nuclear Energy 118 (2018) 291–312

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Numerical methods applied to pin power reconstruction based on coarse-mesh nodal calculation

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ARTICLE INFO

Article history:
Received 10 December 2017
Received in revised form 1 April 2018
Accepted 6 April 2018
Available online 22 April 2018

Keywords:
Pin power reconstruction
Nodal calculation
Finite element methods
Finite difference methods

ABSTRACT

The proposed reconstruction processes combine the two-dimensional (2D) neutron diffusion equation discretized by finite difference method (FDM) and Galerkin finite element method (GFEM) with homogeneous flux distributions. The Finite Element Reconstruction Method (FERM) uses a homogeneous flux distribution over the extremities of each interval that constitute the faces of the fuel assembly (FA). In turns, the Finite Difference Reconstruction Method (FDRM) uses a homogeneous flux distribution over each interval that constitute the faces of the fuel assembly (FA). These flux distributions are obtained for each face of the FA from one-dimensional (1D) polynomial expansions. Such boundary conditions (flux distributions) are based on the average fluxes on the node faces, which are provided by the coarse-mesh nodal calculation performed in homogeneous nodes with dimensions of a FA. The Nodal Expansion Method (NEM) is used for nodal calculation and also provides the multiplication factor of the problem. These reconstruction methods use homogeneous nuclear parameters providing homogeneous flux distributions within the FA. The modulation method is applied to obtain heterogeneous distributions within the FA. To validate the results obtained by the reconstruction methods, such reconstructed heterogeneous distributions are compared with the reference values and with reconstruction performed by the PARCS code. The results show the good accuracy and efficiency of both methods.

1. Introduction

The construction of efficient and safe nuclear power plants requires a prior and accurate determination of the spatial and temporal behavior of the neutron population within nuclear reactor core. This fact has motivated, over the last years, the development of several mathematical methods applied to neutronic calculations. Many of the methods applied to spatial dependence treatment are based on finite difference method (FDM) or finite element method (FEM) (Ackroyd, 1986; Manteuffel and Ressel, 1998; Araujo et al., 2016).

One of the first methods to be used for the spatial discretization of the neutron diffusion equation was the FDM in the early 1950s. Since then, FDM has been widely used in the development of computational codes in the field of Reactor Physics. Due in part to its simple and efficient mathematical formulation. However, FDM requires an excessive refinement of the spatial grid in order to obtain results with acceptable accuracy.

Although FEM emerged in the early 1950s in the field of Structural Engineering, the first application of FEM to nuclear reactor calculations occurred only in 1973 (Kang and Hansen, 1973). In this work, the classical Galerkin finite element formulation (GFEM) was used for fine-mesh calculation. The FEM has become more and more widespread in the scientific community and this is mainly due to its ability to treat non-regular or curved geometries (Hosseini and Derakhshandeh, 2015), in contrast to its main alternative, the FDM, which is limited to only regular geometries. However, the FDM has the advantage of requiring a significantly lower computational effort (virtual memory and/or calculation time), whereas the FEM has greater stability and better convergence rates attainable by the use of higher order elements.

The fine mesh calculations aim to obtaining detailed information about the power distribution within the reactor core. Although such calculations require significant time, both FDM and FEM are suited for this purpose when the goal is to obtain information for reactor design development. However, during the reactor operation, there is the need to perform fast calculations in order to properly adjust the reactivity in the reactor core and also help in the optimization of the refuel patterns. Such a need motivated the development of several coarse-mesh methods that emerged from

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https://doi.org/10.1016/j.anucene.2018.04.010
0306-4549/

the 1970s onwards, among which we highlight the Nodal Expansion Method (NEM).

The coarse-mesh calculations generate accurate results that are required to reactor operation. However, detailed information about the power distribution from the heterogeneity of the reactor is lost during the coarse-mesh calculation. In order to obtain results more quickly and preserve information from the heterogeneity of the reactor, the development of reconstruction methods began in the 1970s.

Different reconstruction methods are found in the literature from the last four decades (Koebke and Wagner, 1977; Koebke and Hetzelt, 1985; Rempe et al., 1988; de Boer and Finnemann, 1992; Joo et al., 2009; Dahmani et al. 2011; Pessoa et al., 2015). These methods differ in how to generate a homogeneous flux distribution which is based on the average values provided by the coarse-mesh nodal method. Such nodal methods differ between solutions using finite difference (Aragones and Ahnert, 1986), analytical solution (Silva et al., 2016), polynomial expansions (Finnemann et al., 1977), semi-analytical solution (Kim et al., 1999) and others. These methods are capable of calculating the multiplication factor, average neutron fluxes and power densities in the reactor's FAs with great precision.

Two reconstruction methods are important for the present work, both use finite difference combined with flux distributions on the FA boundaries. The first method calculates these flux distributions using the maximum entropy (Jung and Cho, 1991) and probability distributions based on the mean values supplied by nodal calculation for obtaining Lagrange multipliers. The second method (Pessoa et al., 2016) determines these distributions using a one-dimensional (1D) fourth order polynomial expansion as a function of the fluxes on the faces and corners of three consecutive FAs, separately for each face of FA. These two reconstruction methods do not use the modulation method, because its solution is given in meshes the size of the fuel cell (pin), making use of heterogeneous cross sections. However, the reconstruction methods presented in this paper use uniform nuclear parameters on the FA area to generate homogeneous distributions within this FA, in which the modulation with form functions is used to determine the heterogeneous distributions.

The main contribution of this work is the use of Galerkin FEM and FDM formulations for spatial discretization of the two group (2G) neutron diffusion equation in two-dimensional domain in order to propose a unified numerical strategy for pin power reconstruction on pressurized water nuclear reactors (PWR), based on homogeneous coarse-mesh calculations. Within the proposed numerical strategy, the first reconstruction method uses finite element (FERM) and the second reconstruction method uses finite difference (FDRM) to spatially discretize the 2D diffusion equation in structured mesh with dimensions of a pin. It is important to emphasize that differently of the reconstruction methods showed in Pessoa et al. (2016, 2018) that are based on heterogeneous nuclear parameters, the strategy proposed in this paper use the same homogeneous nuclear parameters as in the coarse-mesh nodal calculation, in this way, homogeneous distributions are obtained and form functions are used to determine the heterogeneous distributions within the FAs. The use of homogeneous nuclear parameter and form functions is the main difference between the methodology used in the present work, and the methodology used in the last two articles cited.

The numerical discretizations with FDM and Galerkin FEM are combined with a flux distribution on the four faces of a node with dimensions of FA. Such distribution is used as boundary condition for the two proposed solutions. A fourth order 1D polynomial expansion (Pessoa et al., 2016) with five coefficients is used to determine the flux distribution on the four faces of each FA. The coefficients for solution of each face are determined centralizing the solution in one direction on three faces of three consecutive FAs. Thus, three average fluxes on the faces of these FAs and two corner point fluxes between these fluxes on the faces are used as five conditions necessary for the determination of the coefficients.

Nodal Expansion Method (NEM) (Finnemann et al., 1977) is used in this work for coarse-mesh nodal calculation and provides to the reconstruction methods, the average fluxes on the faces of the nodes or FAs and the multiplication factor of the problem.

The corner point fluxes are determined using a third order 1D polynomial expansion with the solution centralized on the four faces of four consecutive nodes and in one direction. This solution is applied to the four corner points of the node separately and the coefficients are determined applying four average fluxes on consecutive faces of four nodes as conditions for the solution. It is important to highlight four points: the first one is that the discontinuity factors on the faces, coming from the homogenization process, are used in coarse-mesh nodal calculation (NEM) and in reconstruction methods; the second one is that the modulation is used; the third one is that the node has the same size as a FA; the fourth one is that the flux distributions on the face of FAs are the minimum set of boundary conditions to be used in the pin power reconstructions.

The remainder of the present work is structured as follows: Section 2 presents the spatial discretization by finite difference and finite element methods of the multigroup neutron diffusion equation in two-dimensional domain. Sections 3 contains a description of the procedures to determine the flux distribution on the four node faces and details about how the corner point fluxes are determined is given in Section 4. Sections 5 and 6 are reserved for the specification of the benchmarks. Section 7 is devoted to description of the methodology used to determine the nuclear power density distribution as well as to the presentation of the numerical results obtained with the reconstruction strategies. Section 8 brings the discussion of all results presented in the previous section. Section 9 contains the conclusions.

2. Discretization of the neutron diffusion equation

In the problems to be solved in this work, the temporal dependence of the neutron diffusion equation is eliminated considering the steady state. Energy dependence is treated from the usual formulation to two energy groups. The spatial dependence, in turn, is discretized using two different numerical methods, namely the finite element method and the finite difference method. The following subsections detail the formulations of the spatial discretization for the FDRM and FERM methods proposed in this paper.

2.1. Spatial discretization by finite elements

The Finite Element Reconstruction Method (FERM) uses the spatial discretization by finite elements of the 2D neutron diffusion equation to obtain a homogeneous flux distribution within the fuel assembly. This discretization is centered on the interface and combined with point fluxes on the vertices of the meshes (pins) constituting the four faces of the node (FA). This boundary condition is calculated using a fourth degree 1D polynomial expansion based on the average quantities provided by coarse-mesh nodal calculation. The NEM provides these average values along with the effective multiplication factor of the treated problem.

Unlike the FDM, the FEM mathematical formulation is not based on the average net currents calculated on the mesh faces (pin), but it is based on the point values of the flux calculated on the vertices of each pin. The development that follows has the objective of detailing discretization by finite elements.