Annals of Nuclear Energy 116 (2018) 10–19

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

General nodal expansion method for multi-dimensional neutronics/thermal-hydraulic coupled problems in pebble-bed core systems

Xiafeng Zhou [a,*], Fu Li [b]

[a] Department of Nuclear Engineering and Technology, School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
[b] Institute of Nuclear and New Energy Technology, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 28 September 2017
Received in revised form 3 December 2017
Accepted 3 February 2018

Keywords:
General nodal expansion method
Multi-dimensional coupled problems
Unified discrete framework
High accuracy on the coarse meshes

ABSTRACT

Motivated by the high efficiency and accuracy of nodal methods on the coarse meshes and the desire to solve the large-scale coupled problems in pebble-bed high temperature gas cooled reactor (HTR), a general nodal expansion method (GNEM) is successfully developed and extended to solve multi-dimensional neutronics/thermal-hydraulic(N/TH) coupled problems in pebble-bed reactor core systems. The developed GNEM combines the respective advantages of traditional nodal expansion methods (NEM) and modified nodal integral methods (MNIM). Specifically, the local analytical solutions of transverse integral equations in MNIM are obtained with the basis function expansion framework and nodal balance equation in NEM. The unified discrete formulation is also developed so that it's convenient for GNEM to solve neutron diffusion equations, convention diffusion equations, pressure poisson equations, heat transfer equations and porous media flow models in the pebble bed core systems. Then all the governing equations of the N/TH coupled problems can be discretized by the GNEM on the coarse meshes so that the computational scale of coupled problems is greatly reduced for the desired precision. To achieve higher accuracy for the coupled problems on the coarse meshes, the high-order information is successfully transferred between the coupled terms by using GNEM. Finally, the coupled GNEM code is formally developed for N/TH coupled problems in pebble-bed reactor core systems. Numerical results show that the coupled GNEM code can track the reference solutions very well on the coarse meshes and show the high-accuracy advantage and the potential of GNEM for complicated multi-dimensional N/TH coupled problems. Further research is needed to add the coupled GNEM to the HTR analysis code-TINTE in order to simultaneously and effectively solve multi-dimensional coupled problems for realistic HTR reactor systems.

1. Introduction

The high temperature gas cooled reactor (HTR) power plant is a large-scale and tight-coupled nonlinear system with multi-physics and multi-loops, even with multiple nuclear steam supplying system modules, which is a huge numerical challenge for high accuracy and efficiency (Wang, 2011). In addition, the current HTR system and safety analysis codes normally use traditional finite volume methods (FVM) and require the fine mesh size to achieve the desired precision and the stable numerical solutions so that the efficiency of these codes is limited for the large-scale coupled problems (Cleveland and Greene, 1986). Therefore, some more efficient numerical methods are urgently developed for complicated HTR coupled problems.

The transverse-integrated nodal methods have been widely used in reactor physics analysis for many years (Lawrence, 1986). They can yield higher numerical accuracy and efficiency on the coarse meshes compared with FVM and finite difference methods (FDM) so that the discrete mesh number, computer storage and computational cost can be greatly reduced for the desired precision. Because of the advantages of transverse-integrated nodal methods, some of them have been extended to the thermal hydraulic calculation. Nodal integral method (NIM) and modified NIM (MNIM) have been developed to solve convection diffusion equations, Burgers equations and Navier-Stokes equations and it's proved that MNIM is more accurate and efficient than the local exact consistent upwind scheme of second order (LECUSSO) (Michael and Dorning, 2001; Wang, 2005; Gunther, 1992). Recently, the traditional nodal expansion method (NEM), a general

* Corresponding author.
E-mail addresses: zhouxiafeng@hust.edu.cn (X. Zhou), lifu@tsinghua.edu.cn (F. Li).

https://doi.org/10.1016/j.anucene.2018.02.009
0306-4549/

NEM (GNEM) and a new nodal expansion method with high-order moments (NEM_HM) have been developed and extended to solve thermal hydraulic problems by our previous research and numerical results show that GNEM and NEM_HM can agree well with the reference solutions, especially for convection-dominated and steep-gradient problems (Zhou et al., 2015, 2016a, 2016b). In addition, the nodal integral expansion method (NIEM) has been also proposed to solve the convection-diffusion equation, but this approach can be only applied for one-dimensional problems now (Lee, 2011). However, the above-mentioned research on nodal methods are mainly about reactor physics analysis or some thermal hydraulic problems, respectively, the nodal methods for neutronics/thermal-hydraulic (N/TH) coupled problems are rarely involved and researched, so some key critical technologies need to be explored.

Therefore, in view of the desire to solve the HTR coupled problems and the high efficiency and accuracy of nodal methods on the coarse meshes, in this paper GNEM is successfully developed and extended to solve more complicated multi-dimensional N/TH coupled problems in pebble-bed reactor core systems. The unified discrete formulation is developed for different governing equations of coupled problems and the high-order information between the coupled terms of different physical fields is successfully transferred using GNEM. The N/TH coupled models are described and analyzed in Section 2. The detailed derivation and some special treatments are presented in Section 3. Some numerical results are shown and discussed in Section 4. Finally, Section 5 gives a brief summary.

2. N/TH coupled models

In this paper the two-dimensional nonlinear N/TH coupled problems in Cartesian geometry include neutron diffusion equations, solid heat transfer models, and mass, momentum and energy equations for porous media models in the pebble-bed core system. All the typical parameters and constitutive equations are chosen or approximated from modular pebble-bed high temperature gas cooled reactor. The specific formulations are presented as follows.

(1) Neutron diffusion equation:

$$-\frac{\partial}{\partial x}\left(D_g\frac{\partial\phi_g}{\partial x}\right)-\frac{\partial}{\partial y}\left(D_g\frac{\partial\phi_g}{\partial y}\right)+\Sigma_g^R\phi_g=Q_g$$
$$=\sum_{\substack{g'=1\\g'\neq g}}^{G}\Sigma^s_{g'\to g}\phi_{g'}+\frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\nu\Sigma^f_{g'}\phi_{g'} \tag{1}$$

where g is the energy group and ϕ_g is the neutron flux for the g-th energy group. D_g, Σ_g^R, $\Sigma^s_{g'\to g}$, $\nu\Sigma^f_{g'}$, χ_g and k_{eff} are the diffusion coefficient, removal, scattering and fission cross-section, neutron fraction and effective multiplication factor, respectively.

(2) Solid heat transfer equation:

$$-\frac{\partial}{\partial x}\left(k_s\frac{\partial T_s}{\partial x}\right)-\frac{\partial}{\partial y}\left(k_s\frac{\partial T_s}{\partial y}\right)=Q \tag{2}$$

where T_s is the solid temperature at core reflector region. k_s is the thermal conductivity and Q the source term.

(3) Fluid flow equations for pebble-bed porous media model:

$$\frac{\partial\rho_f U}{\partial x}+\frac{\partial\rho_f V}{\partial y}=0 \tag{3}$$

$$\rho_f U\frac{\partial U}{\partial x}+\rho_f V\frac{\partial U}{\partial y}-\mu_f\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right)=-\frac{\partial P}{\partial x}+\rho_f\cdot g_x+R_x \tag{4}$$

$$\rho_f U\frac{\partial V}{\partial x}+\rho_f V\frac{\partial V}{\partial y}-\mu_f\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)=-\frac{\partial P}{\partial y}+\rho_f\cdot g_y+R_y \tag{5}$$

where ρ_f is the fluid (helium) density, g_x and g_y are the gravitational accelerations in the different coordinate directions, μ_f is the fluid viscosity coefficient and P is the pressure. U and V are the superficial velocities ($\vec{V}_e=[U,V]^T$). R_x and R_y are the resistance terms for pebble-bed porous media model in the coordinate x and y directions, $\vec{R}=[R_x,R_y]^T$, which are approximated by the Darcy's law as follows:

$$\vec{R}=-K\cdot\vec{V}_e \tag{6}$$

$$K=-k_1\cdot||\vec{V}_e||+k_2 \tag{7}$$

For HTR core, K can be chosen as

$$K=\frac{\psi}{d}\frac{1-\varepsilon}{\varepsilon^3}\frac{\rho_f||\vec{V}_e||}{2} \tag{8}$$

$$\psi=\frac{320}{Re/(1-\varepsilon)}+\frac{320}{[Re/(1-\varepsilon)]^{0.1}} \tag{9}$$

$$k_1=\frac{320}{Re^{0.1}}\frac{\rho_f(1-\varepsilon)^{1.1}}{2d\varepsilon^3} \tag{10}$$

$$k_2=\frac{160\eta_f(1-\varepsilon)^2}{d^2\varepsilon^3} \tag{11}$$

where d is the diameter of fuel sphere and ε is the porosity. Re is the Reynolds number ($Re=\rho_f||\vec{V}_e||d/\eta_f$) and η_f is the dynamic viscosity. When the resistance terms satisfy the following expression,

$$\frac{||\vec{R}||\cdot L}{\rho\vec{V}\cdot\vec{V}}\gg 1 \tag{12}$$

where $\vec{V}$ and L are the characteristic velocity and length, respectively. The influence of convection and diffusion terms can be ignored compared with that of the greater resistance terms. The reference du Toit et al. (2006) presents $||\vec{R}||\cdot L/(\rho\vec{V}\cdot\vec{V})=1470\gg 1$ for HTR pebble-bed core.

(4) Fluid and pebble-bed heat transfer equations:

$$\frac{\partial}{\partial x}\left(\varepsilon\rho_f c_{pf}U\cdot T_f\right)+\frac{\partial}{\partial y}\left(\varepsilon\rho_f c_{pf}V\cdot T_f\right)-\frac{\partial}{\partial x}\left(k_{feff}\frac{\partial T_f}{\partial x}\right)$$
$$-\frac{\partial}{\partial y}\left(k_{feff}\frac{\partial T_f}{\partial y}\right)+\alpha T_f=\alpha T_s \tag{13}$$

$$-\frac{\partial}{\partial x}\left(k_{seff}\frac{\partial T_s}{\partial x}\right)-\frac{\partial}{\partial y}\left(k_{seff}\frac{\partial T_s}{\partial y}\right)+\alpha T_s=(1-\varepsilon)Q_{fiss}+\alpha T_f \tag{14}$$

where T_f and T_s are the fluid and pebble-bed temperature, respectively. α is the heat transfer coefficient between the fluid and pebble-bed. c_{pf}, k_{feff} and k_{seff} are the specific heat at constant pressure, effective fluid thermal conductivity and effective thermal conductivity for pebble-bed core. Q_{fiss} is the thermal source from neutron fission energy.

All the above-mentioned parameters and variables are coupled between each other, which forms a complicated, multi-dimensional, nonlinear coupled N/TH coupled problem.

3. GNEM formulations for multi-dimensional coupled models

Through preliminary analysis, the aforementioned neutron diffusion equations, convention diffusion equations, solid and fluid