



Matrix formulation for the calculation of nuclear reactivity

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ARTICLE INFO

Article history:

Received 8 December 2017

Received in revised form 13 February 2018

Accepted 16 February 2018

Keywords:

Inverse point kinetic equations

Matrix formulation

Nuclear reactivity

Nuclear power reactor

ABSTRACT

In this article, a new method for the calculation of nuclear reactivity using an equivalent matrix function for the concentration of delayed neutron precursors is presented for any given form of nuclear power. The matrix function can be written in a homogeneous system which is solved using an exponential matrix. Its solution requires to integrate the matrix function with respect to time, which originates a second matrix. The exponential form of the resulting matrix is calculated by analytical diagonalization, which reduces the computational cost by avoiding numerical calculation of the eigenvalues, eigenvectors or matrix inversion. The results of the nuclear reactivity calculations show a good precision of the proposed method for different time steps.

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1. Introduction

The processes that take place in a nuclear reactor core are mainly due to the physical phenomenon of nuclear fission. These processes are generated from the reaction of thermal neutrons with heavy atoms such as U-235 and Pu-239. The result of the controlled chain reaction is a great amount of energy which is harnessed by nuclear reactors to generate electrical power (Duderstadt and Hamilton, 1976). In practice, control within the core of these reactors is essentially maintained through a system of reactivity measurements. The precise computational calculation of the nuclear reactivity will guarantee the safe operation of the nuclear reactor, essentially in the ignition tests and when programming the movement of the control rods given a variation of the nuclear power.

The development of digital reactivity meters is based on the inverse equation of point kinetics. Several studies have determined the reactivity with methods that allow the discretization of the integral term associated with this equation, which is known as nuclear power history (Shimazu et al., 1987; Binney and Bakir, 1989; Ansari, 1991; Hoogenboom and Van Der Sluijs, 1988; Kitano et al., 2000; Tamura, 2003). The calculation of nuclear reactivity without this term has also been formulated using derivatives of nuclear power and the least squares method to control noise (Suescún et al., 2007). With the same objective of calculating the nuclear reactivity, other methods such the Discrete Laplace Transform (Suescún et al., 2008), the Euler-Maclaurin Formula (Suescún

et al., 2013) and the three-and five-point formulas of the Lagrange method (Malmir and Vosoughi, 2013) have been implemented.

For the design of real-time reactivity meters, methods that do not involve the history of nuclear power are necessary. Therefore, recent studies have implemented multistep numerical methods for the calculation of nuclear reactivity. Among them, the Hamming's generalized predictor-corrector method (Suescún et al., 2014) and the generalized Adams-Bashforth-Moulton predictor-corrector method (Suescún et al., 2016).

In this paper, an alternative method for the calculation of nuclear reactivity without using the nuclear power history is presented in a simple and accurate manner. The solution is given from the matrix formulation of the concentration of delayed neutron precursors in the equations of the point kinetics with different forms of nuclear power.

2. Theoretical considerations

The point kinetics equations describe the time evolution of neutron density and the concentration of delayed neutron precursors in the reactor core. These equations can be derived from the neutron diffusion equation, resulting in a system of $m + 1$ ordinary, non-linear and strongly coupled differential equations. The mathematical formulation of this system of equations for m groups of delayed neutrons is represented by (Duderstadt and Hamilton, 1976):

$$\frac{dP(t)}{dt} = \left(\frac{\rho(t) - \beta}{\Lambda} \right) P(t) + \sum_{i=1}^m \lambda_i C_i(t), \quad (1)$$

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$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} P(t) - \lambda_i C_i(t), \quad i = 1, 2, \dots, m \quad (2)$$

with the following initial conditions:

$$P(t = 0) = P_0 \quad (3)$$

$$C_i(t = 0) = \frac{\beta_i}{\Lambda \lambda_i} P_0 \quad (4)$$

where $P(t)$ is the nuclear power, $C_i(t)$ is the concentration of the i -th group of delayed neutron precursors ($i = 1, 2, \dots, m$), $\rho(t)$ is the reactivity, Λ is the neutron generation time, β_i is the effective fraction of the i -th group of delayed neutrons, β is the effective total fraction, ($\beta = \sum_{i=1}^m \beta_i$), λ_i is the decay constant of the i -th group of delayed neutron precursors.

It is well known from the literature that the calculation of the reactivity – which depends on the nuclear power form can be determined in an exact way by means of the inverse point kinetics equation (Duderstadt and Hamilton, 1976):

$$\rho(t) = \beta + \frac{\Lambda}{P(t)} \frac{dP(t)}{dt} - \frac{1}{P(t)} \sum_{i=1}^m \lambda_i \beta_i \left[\frac{\langle P_0 \rangle}{\lambda_i} e^{-\lambda_i t} + \int_0^t e^{-\lambda_i(t-t')} P(t') dt' \right] \quad (5)$$

However, this equation can't be applied directly to a real-time reactivity meter due to the history of the nuclear power denoted by the integral term in Eq. (5). Therefore, it is convenient to use Eq. (1) to obtain the reactivity $\rho(t)$:

$$\rho(t) = \beta + \frac{\Lambda}{P(t)} \frac{dP(t)}{dt} - \frac{\Lambda}{P(t)} \sum_{i=1}^m \lambda_i C_i(t), \quad (6)$$

Once $C_i(t)$ is solved numerically from Eq. (2), its solutions are replaced in Eq. (6) to determine the reactivity.

3. Proposed method

The scheme proposed in this section is based on piecewise constant approximation (PCA) for the numerical solution of the equations of point kinetics with a source term (Kinard and Allen, 2004). The PCA method considers this system of non-homogeneous equations in matrix form. Similarly, the set of m differential equations for the concentration of delayed precursors of Eq. (2) can be represented by a non-homogeneous matrix form:

$$\frac{d\vec{\Psi}(t)}{dt} = \mathbf{A}\vec{\Psi}(t)\vec{\mathbf{B}}(t), \quad \vec{\Psi}(0) = \vec{\Psi}_0 \quad (7)$$

where $\vec{\Psi}(t)$ and $\vec{\mathbf{B}}(t)$ are vector functions of dimension m and the $\vec{\Psi}_0$ are the initial conditions given by Eq. (4). Those are expressed as:

$$\vec{\Psi}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \\ \vdots \\ C_m(t) \end{bmatrix}, \quad \vec{\mathbf{B}}(t) = P(t) \begin{bmatrix} \beta_1/\Lambda \\ \beta_2/\Lambda \\ \beta_3/\Lambda \\ \vdots \\ \beta_m/\Lambda \end{bmatrix}, \quad \vec{\Psi}_0 = P_0 \begin{bmatrix} \beta_1/(\Lambda\lambda_1) \\ \beta_2/(\Lambda\lambda_2) \\ \beta_3/(\Lambda\lambda_3) \\ \vdots \\ \beta_m/(\Lambda\lambda_m) \end{bmatrix} \quad (8)$$

Defining $\mathbf{A}$ as a $m \times m$ matrix of constants

$$\mathbf{A} = \begin{pmatrix} -\lambda_1 & 0 & 0 & \dots & 0 \\ 0 & -\lambda_2 & 0 & \dots & 0 \\ 0 & 0 & -\lambda_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda_m \end{pmatrix} \quad (9)$$

The exact solution of the initial value problem given by Eq. (7) can be solved by multiplying this equation by an integrating factor $\exp(-\mathbf{A}t)$ to obtain:

$$\frac{d}{dt} [\exp(-\mathbf{A}t)\vec{\Psi}(t)] = \exp(-\mathbf{A}t)\vec{\mathbf{B}}(t) \quad (10)$$

Then, integrating both sides of Eq. (10) with respect to the time from t_n to t_{n+1} we get:

$$\exp(-\mathbf{A}t_{n+1})\vec{\Psi}_{n+1} - \exp(-\mathbf{A}t_n)\vec{\Psi}_n = \int_{t_n}^{t_{n+1}} \exp(-\mathbf{A}t)\vec{\mathbf{B}}(t)dt \quad (11)$$

where $\vec{\Psi}_{n+1} = \vec{\Psi}(t_{n+1})$ and $\vec{\Psi}_n = \vec{\Psi}(t_n)$. Solving Eq. (11) for $\vec{\Psi}_{n+1}$ we get the following equation

$$\vec{\Psi}_{n+1} = \exp(\mathbf{A}h)\vec{\Psi}_n + \exp(\mathbf{A}t_{n+1}) \int_{t_n}^{t_{n+1}} \exp(-\mathbf{A}t)\vec{\mathbf{B}}(t)dt \quad (12)$$

where $h = t_{n+1} - t_n$ is the time step. From the solution of the inhomogeneous matrix representation given by Eq. (12) it is observed that the second term depends on an integral expression. In this paper, we propose a matrix function of dimension $m+1$ in a homogeneous system that avoids the direct calculation of this integral. This representation is equivalent to the set of m differential equations for the concentration of delayed neutron precursors of Eq. (2) for a given a nuclear power and can be written as follows:

$$\frac{d\vec{\mathbf{x}}(t)}{dt} = \mathbf{S}(t)\vec{\mathbf{x}}(t), \quad \vec{\mathbf{x}}(0) = \vec{\mathbf{x}}_0 \quad (13)$$

where

$$\frac{d\vec{\mathbf{x}}(t)}{dt} = \frac{d}{dt} \begin{bmatrix} P(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_m(t) \end{bmatrix}, \quad \vec{\mathbf{x}}(t) = \begin{bmatrix} P(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_m(t) \end{bmatrix}, \quad \vec{\mathbf{x}}_0 = P_0 \begin{bmatrix} 1 \\ \beta_1/(\Lambda\lambda_1) \\ \beta_2/(\Lambda\lambda_2) \\ \vdots \\ \beta_m/(\Lambda\lambda_m) \end{bmatrix} \quad (14)$$

here $\frac{d\vec{\mathbf{x}}(t)}{dt}$ and $\vec{\mathbf{x}}(t)$ are vector functions of dimension $m+1$ and $\vec{\mathbf{x}}_0$ are the initial conditions given by Eqs. (3) and (4). Thus, the matrix function $\mathbf{S}(t)$ is:

$$\mathbf{S}(t) = \begin{pmatrix} \frac{P'(t)}{P(t)} & 0 & 0 & \dots & 0 \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \dots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \dots & -\lambda_m \end{pmatrix} \quad (15)$$

The matrix function $\mathbf{S}(t)$ has been defined upon the consideration that it must contain the nuclear power $P(t)$ which is known for the calculation of the reactivity. For this reason, the nuclear power $P(t)$ is included in the vector function $\vec{\Psi}(t)$ with its respective initial condition P_0 in $\vec{\Psi}_0$ and the term $\frac{P'(t)}{P(t)}$ for the matrix function $\mathbf{S}(t)$, producing a dimension increase of order $m+1$ that generates the homogeneous representation of the matrix. It can be observed, using Eqs. (13)–(15) exactly how Eq. (2) is reproduced and an equality in the first element that entails $P'(t) = P'(t)$, without producing any contradiction.

Solving Eq. (13) we get:

$$\vec{\mathbf{x}}_{n+1} = \exp\left(\int_{t_n}^{t_{n+1}} \mathbf{S}(t)dt\right)\vec{\mathbf{x}}_n \quad \text{for } n = 0, 1, 2, \dots \quad (16)$$

where $\vec{\mathbf{x}}_n$ is the value of the vector function at a time t_n and $\vec{\mathbf{x}}_{n+1}$ is the value at a later time t_{n+1} . The matrix function $\mathbf{S}(t)$ for the

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