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Measurement errors in degradation-based burn-in

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ABSTRACT

Burn-in is an effective tool to improve product reliability and reduce field failure costs before a product is sold to customers. As many products are becoming highly reliable, traditional burn-in that tests a batch of a product until most weak units fail requires an unaffordable testing duration. If the product failure can be associated with an underlying degradation process and a weak unit degrades faster than a normal one, then degradation-based burn-in can be implemented. Due to such various factors as human errors and limited precision of the measurement device, measurement errors are often inevitable. Ignoring measurement errors in the degradation observations would lead to inferior burn-in decisions. This study uses the Wiener process to model the underlying degradation and considers Gaussian measurement errors in the observations. Two burn-in models with different cost structures are studied and the optimal cutoff level for each model is obtained analytically. The relation between the two models is discussed, leading to a new cost model.

1. Introduction

With the increasing competition in the global market, companies are urged to design and manufacture products with high quality. Nevertheless, it is inevitable that some defects are introduced by the inherent variability of the materials used and the manufacturing process itself [1], leading to a small portion of weak units with high failure rate or fast degradation. The weak units should be identified and scrapped before delivered to customers. Otherwise, losses including warranty costs and reputation will be incurred. Burn-in has been proven to be efficient in identifying and eliminating these weak units. It is also widely adopted in real production.

Many burn-in studies focus on the lifetime distribution of the product and assume a short period of decreasing failure rate of the product [2–8]. With burn-in, the product is operated in conditions similar to field environments so that most early failures will be induced in the test. Accelerated burn-in may be used by elevating the test conditions, such as temperature and voltage, to shorten the test [9,10]. The shock is another method of burn-in to eliminate the weak units [11]. After the test, a useful life period with stable failure rate is reached and the product can function consistently for a relatively long time. From this point of view, burn-in test is a pre-field operation to have the company suffer some disposal costs of weak units instead of administrative and warranty cost from early field failures.

However, as argued by Tseng et al. [12], it becomes more and more unrealistic for today's highly reliable products to fail in a reasonable burn-in duration, even under accelerated test conditions. On the other hand, if a quality characteristic (QC) can properly reveal the status of the product and its degradation is correlated to the failure of the product, it is possible to identify and scrap the weak units by detecting the degradation trend. Based on this argument, some degradation-based burn-in models have been developed in the literature. Wiener processes (especially with linear drift) have been widely used in the product degradation modeling [13–20], and they have been introduced in degradation-based burn-in. For example, Tseng et al. [12] used the Wiener process to model the degradation path and they studied the optimal burn-in strategy based on this degradation model. Ye et al. [21] studied the optimal burn-in strategy based on the Wiener process by considering preventive maintenance during the field operation. Motivated by the fact that the degradation may be not significant, Tseng and Peng [22] applied the integrated Wiener process to model the degradation and discussed the optimal burn-in strategy. Alternatively, some other studies used the Gamma process for the degradation modeling, e.g., [23,24].

The degradation should be measured by human or automatically by some measurement device. In any case, it is inevitable that measurement errors, e.g., due to the inaccuracy of the

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measurement device, would be introduced [16]. Therefore, when designing the burn-in test, it should take measurement errors into consideration. Some researchers have considered measurement errors in studying the degradation data. Whitmore [25] first studied the statistical inference for degradation data by the Wiener process considering measurement errors. Peng and Tseng [13] used the Gaussian distribution to model the unit-to-unit heterogeneity of the degradation rate and proposed a more general Wiener degradation model with measurement errors. Tang et al. [14] utilized the Wiener process with measurement errors in the remaining useful life prediction for the prognostics and health management of Li-ion batteries. Recently, Peng [26] used a mixed Wiener process to account for the heterogeneity in the population and proposed an optimal burn-in strategy for such a product.

The above studies focus much on the degradation data analysis with the Wiener process considering measurement errors, and some of them simultaneously considered the data analysis in the burn-in problem. Undoubtedly, the optimal burn-in strategy depends on the specific cost structure. Therefore, this paper focuses on the optimal burn-in strategies under different cost structures. We consider two different cost models in this paper, i.e., the misclassification cost model and the field failure cost model, and obtain the optimal cutoff levels under the two different cost models. The impact of measurement errors under the first model is further investigated. The relation of the two cost models is discussed and a new model is proposed based on the discussion. The new cost model compromises the two cost models, which approximates the second model but is more analytical tractable.

The remainder of the paper is organized as follows. Section 2 describes the model and the assumptions in the paper. Section 3 discusses the optimal burn-in strategy under the first cost model, i.e., the model considering the misclassification cost of burn-in, and investigates the impacts of measurement errors. Section 4 studies the optimal burn-in strategy under the second cost model, i.e., the model considering the field failure cost. In Section 5, we demonstrate that the two cost models are consistent, and a new cost model is proposed based on the comparison. An example is studied in Section 6 to illustrate the analysis in this paper. Concluding remarks are given in the end.

2. Model description

2.1. Wiener process with linear drift for degradation modeling

Considering a specific QC of the product, the underlying degradation of which, $X(t)$, follows a Wiener process with positive linear drift (probably after some time-scale transformation, as in [12]), i.e.,

$$X(t)=\mu t+\sigma B(t),$$

where $\mu>0$ is the drift rate, $\sigma>0$ is the diffusion coefficient, and $B(t)$ is the standard Brownian motion. As the process has independent and Gaussian distributed increments, i.e., $X(t)-X(u)$ is independent of $X(u)$ for $t>u$ and it follows the Gaussian distribution $N\left(\mu(t-u),\sigma^2(t-u)\right)$, the distribution of the degradation $X(t)$ at fixed time t follows the Gaussian distribution $N\left(\mu t,\sigma^2 t\right)$.

If the pre-determined threshold for $X(t)$ is $X_f>0$, then the first passage time

$$T=\inf\left\{t\ge 0\middle|X(t)\ge X_f\right\}$$

follows an inverse Gaussian distribution with a probability density function (PDF) and a cumulative distribution function (CDF) given by [27]

$$f_{IG}(t;\beta,\lambda)=\left(\frac{\lambda}{2\pi t^3}\right)^{\frac{1}{2}}\exp\left\{-\frac{\lambda(t-\beta)^2}{2\beta^2 t}\right\},$$

and

$$F_{IG}(t;\beta,\lambda)=\Phi\left(\sqrt{\frac{\lambda}{t}}\left(\frac{t}{\beta}-1\right)\right)+\exp\left\{\frac{2\lambda}{\beta}\right\}\cdot\Phi\left(-\sqrt{\frac{\lambda}{t}}\left(\frac{t}{\beta}+1\right)\right),$$

respectively, where $\beta=X_f/\mu$, $\lambda=X_f^2/\sigma^2$, and $\Phi(\cdot)$ is the CDF of the standard Gaussian distribution. Apparently, $T\le t$ indicates that $\max_{0\le s\le t}X(s)\ge X_f$.

2.2. Two subpopulations assumption

Due to defects introduced during manufacturing, e.g., for Micro-Electro-Mechanical Systems (MEMS) [28,29], a fraction of the product would be defective and exhibit fast degradation. Hence, it is common to assume two classes in the product population, i.e., the weak and the normal classes, as in [2,3,11,21]. We assume that degradation of the normal class follows a Wiener process with drift rate $\mu_1>0$ and diffusion coefficient σ, while the weak class degrades following a Wiener process with the same σ but larger drift rate $\mu_2>\mu_1$. The proportions of the normal and the weak classes are $p_1>0$ and $p_2>0$, respectively, where $p_1+p_2=1$.

To identify and eliminate weak units from the main population, a burn-in test is carried out. The duration of the test is denoted by b. After the test, the degradation of each unit is measured and the unit is discarded if the measured degradation is greater than a cutoff level ξ_b. Due to the inaccuracy of the measurement (resulting from human or equipment), it is assumed that the observed degradation is

$$Y_i(t)=X_i(t)+\epsilon, i=1,2$$

where $\epsilon\sim N\left(0,\sigma_\epsilon^2\right)$ is the measurement error and $X_i(t)=\mu_i t+\sigma B(t)$ is the underlying degradation path of the normal $(i=1)$ or the weak class $(i=2)$. It is assumed that the measurement error ϵ is independent of the degradation $X_i(t)$. Therefore, the measured degradation $Y_i(t)$ at a fixed time t follows the Gaussian distribution $N\left(\mu_i t,\sigma^2 t+\sigma_\epsilon^2\right)$.

Burn-in can eliminate weak units and thus influence the life-cycle cost. In the following, we consider two different cost models and study the optimal cutoff level ξ_b and the burn-in duration b under each model. The first cost model (Model 1) involves the cost of misclassification of different classes, while the second cost model (Model 2) considers the field failure cost. It should be mentioned that the related cost parameters are assumed known in this study. For the case where some cost parameters, e.g., the misclassification cost c_α and c_β, are not specified, Wu and Xie [30] proposed a receiver operating characteristic based approach for burn-in decision-making. Interested readers are suggested to refer to Wu and Xie [30] for details.

3. Optimal burn-in test considering misclassification

For products suffering degradation, burn-in can be applied to identify and discard defective units. However, due to the randomness of the QC or measurement errors, normal units may be misclassified as weak while weak units may also muddle through burn-in. Such misclassification would introduce the so-called misclassification cost [12], which contributes to the overall burn-in cost that should be optimized. Such misclassification cost has been considered in burn-in models by e.g., Tseng et al. [12], Tseng and Peng [22] and Tsai et al. [1]. This section studies the optimal burn-in strategy considering the misclassification effect.