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Reactivity determination using the hybrid transport point kinetics and the area method

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ABSTRACT

This paper presents an application of the hybrid transport point kinetic (HTPK) technique to the reactivity determination in subcritical reactor configurations. The mathematical model of the HTPK, initially proposed by Picca et al. (2011) to simulate the time-dependent neutron transport, is here extended to incorporate delayed emissions. The classical area method (Sjöstrand, 1956), developed to invert the point kinetic (PK) model, is then adapted to accommodate the peculiarities of the HTPK approach, to allow its analytical inversion. This novel inverse neutron kinetic methodology is tested on a three-region reactor configuration, showing the interesting performance of the approach based on the HTPK model as compared to the standard area method and highlighting its potential to overcome some of the limitations of the PK-based inversion.

1. Introduction

The point kinetics (PK) model is known to provide reasonable approximations of the thermal power evolution in a reactor starting from criticality (Henry and Curlee, 1958). This is due to the fact that the initial equilibrium between neutron density and precursor concentrations helps mitigating the shape evolution (i.e. spatial and energy neutron distribution), at least on a short timescale as compared with the characteristic time of the delayed emission. Under this condition, using a lumped parameter model (e.g. the PK) becomes very effective as it dramatically reduces the computational burden for transient evaluations.

When considering source pulsed experiments in subcritical systems, the validity of the point kinetic basic assumption becomes questionable. As an example, in experiments in Accelerator-Driven Systems (Salvatores et al., 1996; OECD/NEA, 2002), the source transients are often initiated from non-equilibrium states between neutrons and precursors and, as a result of that, the neutron shape distortions are generally quite significant also on shorter timescale. Additionally, the high-energy neutrons generated by the external source typically induce strong spectral and spatial effects, which are difficult to account for considering the fixed shape assumption as in the PK.

Although several kinetics methods were proposed for ADSs, including the quasi-static (Devooght, 1980) and the multi-point kinetics (Kobayashi, 1992; Ravetto et al., 2004), their application to there activity determination problem has always been constrained by limited availability of analytical inverse solution methods and by the capability to interpret the results of the inversion. More recently, an alternative approach was proposed by Picca and Furfaro (2012, 2017) who developed model inversion methods based on both Neural Networks (NN) and Gaussian Processes (GP). Although these methodologies enable the use of more sophisticated kinetic models, hence reducing the *model error* in the reactivity estimation, they do not allow a direct analytical inversion which is generally possible with other methods derived from the PK (e.g. area method, Sjöstrand, 1956). Consequently, NN and GPs based approaches exhibit limitations in controlling of the *inversion error*, although approaches to limit it were proposed in (Picca and Furfaro, 2012).

In this paper, we develop an inverse methodology that employs the Hybrid Transport Point Kinetics (HTPK) to determine the reactivity of a multiplying system. Proposed by Picca et al. (2011, 2012), the HTPK is generally more accurate than the PK for the modelling of the source-driven problems (i.e. in the limit of truncation order going to infinity, it reproduces the full transport solution). Because the model for the residue in the HTPK is based on a system of first order ODEs similar to the PK (i.e. the HTPK reduces to PK at its order zero), with suitable modifications its inversion can be approached using classical techniques such as the area method (Sjöstrand, 1956). By developing an analytical inversion

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of the HTPK for the reactivity determination, one aims at generating inverse solutions that are more accurate than PK-based solutions and that can be refined utilizing the truncation order in the HTPK.

In this work, after a brief review of the fundamental reactor kinetic equations (Section 2), the HTPK is extended to include the neutron delayed emissions (Section 3) and combined with the classical area method to accommodate the peculiarity of the HTPK model (Section 4). The performances of this variant of the area method are tested on a test casein a 1D Cartesian geometry and compared against the classical area method results (Section 5), before drawing some conclusions in Section 6.

2. Reactor kinetics equations

The time-dependent linear Boltzmann equation for monoenergetic particles in subcritical media and Cartesian geometry can be written as follows (Akcasu et al., 1971):

$$\begin{cases} \frac{1}{\upsilon}\frac{\partial}{\partial t}\phi(\vec{x},\vec{\Omega},t)+\mathbf{L}(\vec{x})\phi(\vec{x},\vec{\Omega},t)=[\mathbf{M}_s(\vec{x})+(1-\beta_{tot})\mathbf{M}_f(\vec{x})]\phi(\vec{x},\vec{\Omega},t) \\ \qquad\qquad +\sum_{n=1}^{N}\frac{\lambda_n}{4\pi}C_n(\vec{x},t)+\frac{1}{4\pi}q_{ext}(\vec{x},t) \\ \frac{\partial}{\partial t}C_n(\vec{x},t)=-\lambda_n C_n(\vec{x},t)+4\pi\beta_n\mathbf{M}_f(\vec{x})\phi(\vec{x},\vec{\Omega},t) \quad \text{with} n=1,\ldots,N \end{cases} \tag{1}$$

where $\beta_{tot}=\sum_{n=1}^{N}\beta_n$ and, in case of isotropic scattering, the following definitions holds:

$$\mathbf{L}(\vec{x},\vec{\Omega})=\vec{\Omega}\cdot\nabla+\Sigma(\vec{x})\mathbf{M}_s(\vec{x})=\frac{\Sigma_s(\vec{x})}{4\pi}\oint_{4\pi}\cdot d\vec{\Omega}\prime\mathbf{M}_f(\vec{x}) =\frac{\upsilon\Sigma_f(\vec{x})}{4\pi}\oint_{4\pi}\cdot d\vec{\Omega}\prime \tag{2}$$

Although in general the cross sections may depend on the time variable, note that the mathematical operators in Eqs. (2) are time-independent as only source-driven transients are considered in this paper. Considering zero neutron flux before the source pulse and no initial precursor build-up, the boundary and initial conditions for Eqs. (1), (2) are:

$$\phi(\vec{x}_S,\vec{\Omega},t)=0, \text{for } \vec{\Omega}\cdot\vec{n}_S<0\phi(\vec{x},\vec{\Omega},0)=0 \tag{3}$$

and:

$$C_n(\vec{x},0)=0 \tag{4}$$

In Eq. (3), $\vec{x}_S$ defines the boundaries of the medium and $\vec{n}_S$ is the outward-directed surface normal.

It is well-known that solving of the time-dependent problem in Eqs. (1)–(4) is numerically challenging. This is mainly due to the disparity between the neutron and precursor time constants, which typically differ by several orders of magnitude. The PK model represents a widely-used approximation of the model in Eq. (1) for coupled evolution of neutrons and precursor concentrations. Whilst the PK model works well for initially critical systems, its model approximating capabilities degrades when dealing with source-driven transients, where strong neutron flux distortions are typically observed. In this case, the assumption of a constant neutron shape across the transient does not rigorously hold. As discussed in Picca et al. (2011, 2012), the HTPK represents a powerful alternative to the PK in that it improves its performance without significantly affecting the overall computational time.

In the following section, the fundamentals of the HTPK model are reviewed and extended to the general case with delayed emissions, not addressed in previous papers by Picca et al. (2011, 2012).

3. Hybrid transport point kinetic method

The starting point for the derivation of the HTPK is a multi-collision approach to the solution of the linear Boltzmann equation, i.e.:

$$\phi(\vec{x},\vec{\Omega},t)=\sum_{j=0}^{\infty}\phi^{[j]}(\vec{x},\vec{\Omega},t) \tag{5}$$

Each contribution in the sum in Eq. (5) is the solution of the following equations:

$$j=0:\frac{1}{\upsilon}\frac{\partial}{\partial t}\phi^{[j]}(\vec{x},\vec{\Omega},t)+\mathbf{L}\phi^{[j]}(\vec{x},\vec{\Omega},t)=\frac{1}{4\pi}q_{ext}(\vec{x},t) \tag{6a}$$

$$j>0:\frac{1}{\upsilon}\frac{\partial}{\partial t}\phi^{[j]}(\vec{x},\vec{\Omega},t)+\mathbf{L}\phi^{[j]}(\vec{x},\vec{\Omega},t) =[\mathbf{M}_s+(1-\beta_{tot})\mathbf{M}_f]\phi^{[j-1]}(\vec{x},\vec{\Omega},t) \tag{6b}$$

The idea behind the HTPK equations is to truncate the series in Eq. (5) at the order T and use an approximate model for the residue (Picca et al., 2011, 2012):

$$\phi(\vec{x},\vec{\Omega},t)=\sum_{j=0}^{T}\phi^{[j]}(\vec{x},\vec{\Omega},t)+r(\vec{x},\vec{\Omega},t) \tag{7}$$

The mathematical nature of the equation for the residue is the neutron balance equation in Eq. (1), i.e.:

$$\frac{1}{\upsilon}\frac{\partial}{\partial t}r(\vec{x},\vec{\Omega},t)+\mathbf{L}r(\vec{x},\vec{\Omega},t)=[\mathbf{M}_s+(1-\beta_{tot})\mathbf{M}_f]r(\vec{x},\vec{\Omega},t)+[\mathbf{M}_s +(1-\beta_{tot})\mathbf{M}_f]\phi^{[T]}(\vec{x},\vec{\Omega},t) +\sum_{n=1}^{N}\frac{\lambda_n}{4\pi}C_n(\vec{x},t) \tag{8}$$

Boundary and initial conditions for Eqs. (6a), (6b) and (8) are the same as in Eqs. (3). In the HTPK formalism, the equation for the precursor concentrations can be written as:

$$\frac{\partial}{\partial t}C_n(\vec{x},t)=-\lambda_n C_n(\vec{x},t)+4\pi\beta_n\mathbf{M}_f\left[\sum_{j=0}^{T}\phi^{[j]}(\vec{x},\vec{\Omega},t)+r(\vec{x},\vec{\Omega},t)\right] \tag{9}$$

with the initial condition as in Eq. (4).

Following the HTPK approach in Picca et al. (2011), the residue in Eq. (8) can be approximated via a lumped parameter model similar to the PK system of equations, factorizing the solution as:

$$r(\vec{x},\vec{\Omega},t)\approx\psi(\vec{x},\vec{\Omega})a(t) \tag{10}$$

where $\psi(\vec{x},\vec{\Omega})$ is the shape and $a(t)$ the amplitude. Considering the following definition for the reference problem for the shape (Picca et al., 2012):

$$[\mathbf{L}-\mathbf{M}_s-\mathbf{M}_f]\psi(\vec{x},\vec{\Omega}) =[\mathbf{M}_s+\mathbf{M}_f]\phi_0^{[T]}(\vec{x},\vec{\Omega})[\mathbf{L}^{+}-\mathbf{M}_s^{+}-\mathbf{M}_f]\psi^{+}(\vec{x},\vec{\Omega})=\upsilon\sum_f \tag{11}$$

the point model for the residue and the precursor concentration can be written as:

$$\begin{cases} \left\langle\psi^{+},\frac{1}{\upsilon}\psi\right\rangle\frac{da(t)}{dt}+\left\langle\psi^{+},[\mathbf{L}-\mathbf{M}_s-(1-\beta_{tot})\mathbf{M}_f]\psi\right\rangle a(t) \\ \quad =\sum_n\frac{\lambda_n}{4\pi}\left\langle\psi^{+},C_n(\vec{x},t)\right\rangle+\left\langle\psi^{+},[\mathbf{M}_s-(1-\beta_{tot})\mathbf{M}_f]\phi^{[T]}\right\rangle \\ \frac{d\left\langle\psi^{+},C_n(\vec{x},t)\right\rangle}{dt}=-\lambda_n\left\langle\psi^{+},C_n(\vec{x},t)\right\rangle+4\pi\beta_n\left\langle\psi^{+},\mathbf{M}_f\psi\right\rangle a(t) \\ \quad +\left\langle\psi^{+},4\pi\beta_n\mathbf{M}_f\sum_{j=1}^{T}\phi^{[T]}\right\rangle \quad \text{with } n=1,\ldots,N \end{cases} \tag{12}$$

where $\langle f,g\rangle\equiv\int_V f(\vec{x})g(\vec{x})d\vec{x}$. Introducing the following definitions: