



# Quantification of control rod worth uncertainties propagated from nuclear data via a hybrid high-order perturbation and efficient sampling method

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## ABSTRACT

Based on the high order perturbation theory and few-group neutron diffusion equation, the formula to evaluate control rod worth is derived for various core state, which is equivalent to establish a function between CRW and eigenvalue, few-group homogenized cross sections, flux and adjoint flux. Then, an effective hybrid high order perturbation and efficient sampling (HOPES) method to quantify the uncertainty of control rod worth propagated from uncertainties in input parameters is proposed. To verify the validity of the HOPES method, a three dimensional mini core model with typical AP1000 fuel assemblies in a  $3 \times 3$  checkerboard pattern is built and uncertainty in nuclear data is chosen as the main uncertainty source. Numerical results indicate that the HOPES method is much more effective to directly propagate uncertainties in input parameters to the final control rod worth and can quantify the uncertainty of either differential or integral control rod worth more accurately. Finally, the uncertainty of differential and integral CRW propagated from nuclear cross sections was quantified for the first time by using the HOPES method.

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## 1. Introduction

Nuclear reactor core design and safety analysis both need an accurate calculation of control rod worth (CRW), which can provide precise and adjustable control of reactivity and specify safety margin of the nuclear reactor. However, uncertainty inevitably exists in the CRW calculation. Here, the uncertainty in the calculated CRW is equivalent to the standard deviation based on measurements that are made as part of startup tests. In general, the calculated CRWs are in agreement with measured bank worths in the range of 0–10% (Diamond, et al., 2000). For quantifying uncertainty of CRW, there are two major uncertainty sources need to be studied in-depth. First, uncertainty exists in input parameters, such as multi-group cross sections, fission product yields and decay data for burnup calculation, geometry, material composition, enrichment variations and thermal-hydraulic quantities. Second, calculation models as well as methods also have a large contribution to the total uncertainty of CRW. As a result of all these uncertainty sources, the total uncertainty of CRW is large and anything

over 15% would be unacceptable (Diamond, et al., 2000). However, the uncertainty is considered more qualitative than quantitative. In addition, the accurate CRW uncertainty and the contribution of each factor to the total uncertainty is unknown, especially for the new types of nuclear reactor.

Recently, many effort have been put into uncertainty quantification of  $k_{\text{eff}}$  and power distribution of different types of reactor core, such as the project for the uncertainty analysis for the modeling of Light Water Reactor (LWRs) supported by OECD (Ivanov et al., 2013) and the IAEA CRP project for uncertainty analysis of High Temperature Gas Cooled Reactor (HTGR) (Reitsma et al., 2012). But there is no systematic study to quantify the contribution of each uncertainty source to the total uncertainty of CRW up to now. Although some general uncertainty analysis methods can be used, such as perturbation theory (Williams, 1986) and statistical sampling method (Helton et al., 2006), and experiences can be learnt from OECD LWR UAM and IAEA CRP project, uncertainty analysis of CRW is still full of challenge due to the particularity and complexity of CRW calculation. But understanding uncertainty in CRW is important for improving the reliability of the numerical results, identifying the importance of uncertainty sources and deciding where additional efforts should be undertaken to reduce

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uncertainties and to improve the design. Therefore, CRW uncertainty analysis is a good topic worthy to investigate as soon as possible.

Quantifying the uncertainty of CRW should start from the evaluation of CRW. In general, control rod worth is evaluated by the traditional twice critical method, that is CRW is calculated by the difference between reactivity of the core with varying insertion depths. The uncertainty of CRW can be quantified based on the bivariate variance analysis theory. However, there is a strong correlation between various reactivity when the control rod is withdrawn, due to the fact that the core has same assembly arrangement and similar material components and cross sections are also correlated. And the uncertainty of CRW is quite sensitive to the correlation between different reactivity. Therefore, the correlation coefficient should be first quantified accurately, which is quite difficult. An alternative method to evaluate the uncertainty of CRW is using sampling statistical method, by which the issue of high correlation can be reasonably addressed. However, a strong convergence accuracy of eigenvalue is needed to predict uncertainty of differential CRW more accurately if traditional twice critical method is used to propagate uncertainty. For small perturbation, high order perturbation theory can be used to predict the differential CRW when the control rod is withdrawn in small increments. In this way, the formula to evaluate CRW is derived for various core state by using the high order perturbation theory and few-group neutron diffusion equation, which is equivalent to establish a function between CRW and eigenvalue, few-group homogenized cross sections, flux and adjoint flux. Then, an effective hybrid high order perturbation and efficient sampling (HOPES) method to quantify the uncertainty of CRW propagated from uncertainties in input parameters is proposed in this paper. This method is valid very generally for propagating uncertainties of different input parameters to CRW and quantifying its uncertainty, as long as these uncertainty sources can be propagated to the macroscopic cross sections, such as uncertainties in multi-group cross sections, fission product yields and decay data for burnup calculation, geometry, material composition, enrichment variations and so on. In this paper, the main uncertainty source is nuclear cross sections. To verify the validity of the HOPES method, a three dimensional mini core model with typical AP1000 fuel assemblies in a  $3 \times 3$  checkerboard pattern is built and the uncertainty of differential and integral CRW propagated from nuclear cross sections was quantified.

In the following sections, the derivation of CRW by using high order perturbation theory and the new proposed HOPES method are presented in Section 2. Section 3 shows the details of mini core model and basic few-group homogenized cross sections, uncertainty and sensitivity information for this model calculated through NEWT, Tsunami-2d and Tsunami-IP modules in SCALE6.1 (SCALE, 2011). In Section 4, numerical experiments are given to illustrate the validity of the new proposed HOPES method and the uncertainty of CRW propagated from nuclear data is also quantified. Finally, some conclusions are drawn in Section 5.

## 2. Methodology

### 2.1. High-order perturbation theory to predict control rod worth

Based on the three-dimensional (3D) neutron diffusion theory, the differential CRW is a function of assembly homogenized cross sections, neutron flux and adjoint flux, which can be predicted by using high-order perturbation theory when the control rod is withdrawn in small increments. The summation of all the differential CRW further gives the integral CRW. The differential CRW is eval-

uated by high order perturbation theory as follows. The  $\lambda$ -mode Boltzmann equation represented using operator notation is given as:

$$M\phi = \frac{1}{k}F\phi \quad (1)$$

where,  $F$  and  $M$  are the fission and “absorption-and leakage” operators. The equation adjoint to Eq. (1) is

$$M^*\phi^* = \frac{1}{k}F^*\phi^* \quad (2)$$

where,  $F^*$  and  $M^*$  are the adjoint operators for  $F$  and  $M$ .  $\phi^*$  is the adjoint flux.  $k$  is the effective multiplication factor for the unperturbed system. Then, a new operator  $Q$  is defined as:

$$Q = M - F \quad (3)$$

Based on the definition of reactivity, we have

$$\rho = \frac{k - 1}{k} \quad (4)$$

Then, Eq. (1) can be rewritten as

$$Q\phi = -\rho F\phi \quad (5)$$

And the equation adjoint to Eq. (5) is given as:

$$Q^*\phi^* = -\rho F^*\phi^* \quad (6)$$

Suppose that the control rod is withdrawn in small increments, which is just like a perturbation to the core, so that the elements of  $F$  and  $M$  are changed. Then the equation for the perturbed reactor can be written as

$$Q'\phi' = -\rho'F'\phi' \quad (7a)$$

$$Q' = Q + \delta Q \quad (7b)$$

$$F' = F + \delta F \quad (7c)$$

where  $Q'$ ,  $\rho'$ ,  $F'$  and  $\phi'$  denote the parameters of perturbed reactor and  $\delta Q$  and  $\delta F$  are the change due to the perturbation.  $\rho' = (k' - 1)/k'$  and  $k'$  is the effective multiplication factor for the perturbed system. The reactivity of the perturbed reactor can be rewritten as:

$$\rho' = \rho + \rho_c \quad (8)$$

where,  $\rho_c$  is the change of reactivity due to the movement of control rod. To obtain the perturbed flux and reactivity, the iterative method should be used, in which the values  $\delta Q$  and  $\delta F$  in Eq. (7) is replaced by  $\tau\delta Q$  and  $\tau\delta F$  respectively. Then Eq. (7) is changed into the following form:

$$(Q + \tau\delta Q)\phi' = -\rho(F + \tau\delta F)\phi' - \rho_c(F + \tau\delta F)\phi' \quad (9)$$

For solving Eq. (9), the perturbed flux and reactivity are expressed in the following power series:

$$\phi' = \phi^{(0)} + \tau\phi^{(1)} + \tau^2\phi^{(2)} + \dots \quad (10a)$$

$$\rho_c = \tau\rho^{(1)} + \tau^2\rho^{(2)} + \tau^3\rho^{(3)} + \dots \quad (10b)$$

where,  $\phi^{(n)}$  and  $\rho^{(n)}$  correspond to the  $n$ th order perturbed flux and reactivity respectively. If  $\tau$  is equal to unity in Eq. (10),  $\phi'$  and  $\rho_c$  become equivalent to the solutions of Eq. (7) (Mitani, 1973). Substituting Eq. (10) into Eq. (9) and equating the coefficients of equal powers of  $\tau$  on both sides of the resultant equation, a series of equations which express the high orders of perturbation can be obtained as following:

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