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An Adjoint Proper Orthogonal Decomposition method for a neutronics reduced order model

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ABSTRACT

This paper deals with the use of Reduced Order Methods for neutronics modelling. This approach is used whether both accuracy and computational efficiency are required. A very popular category of these methods is based on projection approaches which use spatial basis and test functions for the development of the reduced order model. The selection of the spatial basis and test functions used in the projection phase is a crucial issue since it has an impact on the accuracy and the computational cost. In this work, different methods for the creation of the spatial basis and the test functions are analysed. In particular, an Adjoint Proper Orthogonal Decomposition (APOD) method is proposed combining the properties of the Proper Orthogonal Decomposition and the use of the adjoint flux as test function in the neutronics framework. The different methods are applied to create a spatial neutronics model for the ALFRED reactor. The simulation results show that the APOD method gives better results compared to the other methods (Modal Method and standard Proper Orthogonal Decomposition) increasing the accuracy of the reduced order model or minimizing the computational cost.

1. Introduction

Different approaches can be found for the neutronics modelling with different degrees of accuracy and related computational effort, e.g., Monte Carlo method, deterministic transport theory, diffusion approximation, Point Kinetics (PK). The selection of the proper approach, according to the specific simulation and analysis needs, is crucial. In this regard, the recent developments in the simulation of nuclear reactors, along with the increased availability of computational resources, do not cancel the importance of having tools that provide useful insights at reasonable computational time. The adoption of Reduced Order Methods (ROMs) (Hesthaven et al., 2016; Rozza et al., 2008) can be suitable for this aim, especially in the areas of process optimization, control or uncertainty quantification (Chinesta et al., 2016; Gunzburger, 2002; Quarteroni et al., 2011). The interest in Reduced Order Methods for the simulation of complex systems in nuclear field is increased in the last years, being applied to Monte Carlo methods (Aufiero et al., 2016; Aufiero and Fratoni, 2017), to the deterministic transport equations (Bang et al., 2015; Buchan et al., 2015), and diffusion problems (Buchan et al., 2013; Gong et al., 2016; Lorenzi et al., 2015; Sartori et al., 2014). The fields of application of ROMs are not limited to the neutronics modelling but they are employed also in sensitivity analysis and uncertainty quantification (Abdel-Khalik et al., 2013; Bang et al., 2012a) and in thermal–hydraulic context (Lorenzi et al., 2017, 2016). Different from the Surrogate Response Surfaces (SRS) approach which are based on data fitting, the methods employed in this work belong to the projection-based family. These Computation Reduction Techniques (CRT) are aimed at reducing the dimension of the algebraic system through the projection onto a small subspace made by global basis functions (Manzoni et al., 2012). As for the neutronics, this paradigm can be applied separating the spatial and time dependence of the neutron flux. The latter can be represented as a linear combination of spatial basis functions calculated from an accurate neutronics modelling weighted by time-dependent coefficients. The dynamic behaviour of the flux is reduced to the study of these time-dependent coefficients, and can be represented by a set of Ordinary Differential Equations (ODEs). This set is obtained multiplying the governing Partial Differential Equations (PDEs) with suitable test functions, as in Galerkin methods. The selection of the spatial basis and test functions is a crucial issue in the development of the reduced order model. In particular, the aim is selecting the optimal pair of spatial basis/test functions that maximizes the accuracy of the model and minimizes the computational cost.

In this work, different approaches in the calculation of both the spatial basis and the test functions are assessed in terms of efficiency (considered as the ratio between accuracy and

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List of symbols

Latin Symbols

A	coefficient used in Eq. (11), cm^{-1}
C_j	concentration of the j^{th} precursor group, cm^{-3}
D_g	neutron diffusion coefficient of the g^{th} energy group, cm
E	energy, MeV
E_g	energy group threshold, eV
G	number of energy group, –
h_{CR}	height of control rods, m
N	number of employed functions in the spatial basis, –
N_s	number of snapshots in the POD method, –
N_p	number of employed POD functions for the spatial basis, –
$\boldsymbol{n}$	surface normal unit vector, –
n_i^g	time-dependent coefficient of the i^{th} spatial function of the neutron flux of the g^{th} energy group, –
$\boldsymbol{r}$	spatial coordinate, cm
S	surface of the spatial domain, cm^2
T	temperature, K
t	time, s
v_g	neutron speed of the g^{th} energy group, cm s^{-1}

Greek Symbols

β	total delayed neutron fraction, pcm
β_j	delayed neutron fraction of the j^{th} precursor group, pcm
γ	albedo coefficient used in Eq. (12), –
λ_j	decay constant of the j^{th} precursor group, s^{-1}
λ_i^*	i^{th} eigenvalue, –
ν	average number of neutrons emitted per fission event, –
ξ_i^g	i^{th} test function of the g^{th} energy group, –
ρ	reactivity, pcm
Σ	generic macroscopic cross-section, cm^{-1}
Σ_a^g	macroscopic absorption cross-section of the g^{th} energy group, cm^{-1}
Σ_f^g	macroscopic fission cross-section of the g^{th} energy group, cm^{-1}
$\Sigma_s^{g\rightarrow}$	macroscopic cross-section including scattering out of the energy group g, cm^{-1}
$\Sigma_s^{g\rightarrow g'}$	macroscopic group transfer cross-section from energy group g to g′, cm^{-1}
ϕ_g	neutron flux of the g^{th} energy group, cm^{-2} s^{-1}
χ_d^g	fraction of delayed neutrons generated in the g^{th} energy group, –
χ_p^g	fraction of prompt neutrons generated in the g^{th} energy group, –
χ_t^g	fraction of total neutrons generated in the g^{th} energy group, –
ψ_i^g	i^{th} spatial eigenfunction of the neutron flux of the g^{th} energy group, cm^{-2} s^{-1}
Ω	spatial domain, cm^3

Subscripts

0	reference value
a	axial
f	fuel
g	energy group number
gz	generic zone
r	radial

computational time). As for the spatial basis calculation, the classic option is the Modal Method (MM) that employs as spatial basis the eigenfunctions of the neutron diffusion PDEs calculated in a reference configuration (Stacey, 1969). The Modal Method provides a "general" spatial basis since it is related to the eigenvalue problem of the system, referring to a reference configuration. On the other hand, the natural alternative is to build an *ad-hoc* spatial basis tailored on specific simulations. This is the case of the Proper Orthogonal Decomposition (POD) with the snapshot technique (Holmes et al., 1996; Sirovich, 1987). In this case, some proper solutions of the neutron diffusion PDEs are calculated (i.e., the "snapshots") and the most relevant modes are selected.

As for the test functions, the classic option is to use the same functions that constitute the spatial basis as in a Galerkin projection. Nevertheless, the test functions can be different as in a Petrov-Galerkin projection. In this work, an Adjoint Proper Orthogonal Decomposition approach has been developed to combine the properties of the Proper Orthogonal Decomposition and the use of the adjoint flux as test function in the neutronics framework.

The different approaches are applied to build a spatial neutronics model of the Advanced Lead Fast Reactor European Demonstrator (ALFRED) (Alemberti et al., 2014). The correct reproduction of the reactivity insertion following a temperature or a Control Rod (CR) change is the main quantity of interest in our case. In this regard, the study of a Lead Fast Reactor represents an interesting case study since in this nuclear system the impact of the coolant density variation may act in different directions (i.e., with a positive or negative local coefficient) according to the core zone involved. This spatial feature cannot be captured with a simplified neutronics modelling (i.e., the Point Kinetics), and a reduced order model can be the desired compromise between accuracy and computational time.

The paper is organized as follows. In Section 2, the ALFRED plant layout and the core configuration are briefly introduced. In Section 3, the modelling approach employed for the spatial neutronics model is presented with the description of the several phases involved in the procedure, with a particular attention to the spatial basis and test functions selection. The simulation results of the ALFRED full core modelling are presented in Section 4, analysing the optimal spatial basis/test functions pair and the performance of the several methods in assessing the reactivity following a temperature change and a CR movement. Finally, some conclusions, perspectives and further developments are given in Section 5.

2. The Advanced Lead Fast Reactor European Demonstrator

The reference reactor in this work is the Advanced Lead-cooled Fast Reactor European Demonstrator (ALFRED), developed within the European FP7 LEADER Project. The Project efforts were mainly focused on the resolution of the key issues that emerged in the previous Euratom ELSY Project (Cinotti et al., 2008) to reach a new reference reactor configuration, which was used to design a fully representative scaled-down prototype. The demonstration ALFRED unit is expected to be built at ICN (Institute de Cercetari Nucleare) facility near Pitesti in southern Romania (Alemberti et al., 2013a).

ALFRED is a small-size (300 MW_{th}) pool-type LFR. The current configuration of its primary system (Alemberti et al., 2013b) is depicted in Fig. 1. All the major reactor primary system components, including core, primary pumps, and Steam Generators (SGs), are contained within the reactor vessel, being located in a large lead pool inside the reactor tank. The coolant flow coming from the cold pool enters the core and, having passed through the latter, is collected in a volume (hot collector) to be distributed to eight parallel pipes and