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Artificial neural network approximations of linear fractional neutron models

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ABSTRACT

In this paper the artificial neural network (ANN) approximation for fractional neutron point kinetics (FNPK) model and fractional reduced-order model (F-ROM) is presented. The input-output data of the step response generated using the closed-loop stable fractional-order models was used for the training the ANN models. The ANN topology with various layers and different number of neutrons was tried to obtain the best approximation for the fractional-order model. The results confirm that the designed ANNs provide a good approximation to linear FNPK and F-ROM models. It was also observed that the convergence and learning of ANN is greatly affected by the type of model (FNPK or F-ROM), value of anomalous diffusion coefficient (fractional-order derivative) and the relaxation time.

1. Introduction

Recent years have witnessed the rapid development of applications of fractional calculus, such as calculus with derivatives and integrals with arbitrary real or complex order (see Podlubny, 1999) in various field like control systems (Axtell and Bise, 1990), signal processing (Assaleh and Ahmad, 2007), physics (Wu and Shen, 2009), Economics (Tejado et al., 2017), etc. The field of nuclear reactor modeling (Duderstadt and Hamilton, 1976) is also not an exception to this. Fractional derivative operators have been successfully employed to model the process of neutron transport inside the reactor core (e.g., Espinosa-Paredes et al., 2008; Vyawahare and Nataraj, 2013a,b). The fundamental motivation for this approach is to consider the movement of neutrons as anomalous diffusion, particularly subdiffusion (Metzler and Klafter, 2000). The fractional-order models for nuclear reactor can be broadly classified into various different categories depending on the process considered, viz, modeling of neutron transport as anomalous diffusion with time fractional derivative (Espinosa-Paredes and Polo-Labarrios, 2012; Vyawahare and Nataraj, 2013a, b), modeling of neutron transport with space fractional derivative (Espinosa-Paredes et al., 2013; Moghaddam et al., 2014; 2015a,b, c; Espinosa-Paredes, 2017), modeling of fractional-order neutron point kinetics equations (FNPK models) (Espinosa-Paredes et al., 2011; Vyawahare and Nataraj, 2015), development of linear FNPK models and their stability analysis (Cázares-Ramírez et al., 2017; Vyawahare and Espinosa-Paredes, 2017). These models are claimed to provide a more accurate and faithful representation to nuclear reactor dynamics and are more compact as compared to their integer-order (IO) counterparts. These models have been analyzed in detail for numerical computation and stability linear.

This paper presents the artificial neural network (ANN) approximations for two fractional models:

- FNPK: Linear fractional neutron point kinetics model for reactor with unity feedback (Cázares-Ramírez et al., 2017), and
- F-ROM: Linear fractional reduced-order model with unity feedback (Vyawahare and Espinosa-Paredes, 2017).

For both the FNPK and F-ROM models, the relaxation time constants are considered as follows: long relaxation time, $\tau = 1 \times 10^{-3}$ s(τ_1) and short relaxation time, $\tau = 1 \times 10^{-5}$ s(τ_2). Development and rigorous stability analysis of these two models of fractional order were presented in the work of Cázares-Ramírez et al. (2017) and Vyawahare and Espinosa-Paredes (2017). These models were developed from the original nonlinear FNPK and F-ROM equations were represented in terms of fractional-order transfer functions. The three values of anomalous diffusion coefficient (fractional-order derivatives) considered are given in Table 1.

Combination of these values with the two relaxation constants represents all possible situations of the neutron movement dynamics inside the heterogeneous reactor core and captures all

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Table 1
Fractional-orders (α) for FNPK and F-ROM models.

α	Value	Physical interpretation
α_1	0.25	High subdiffusivity
α_2	0.5	Medium subdiffusivity
α_3	0.8	Low subdiffusivity

the features of both the fractional-order models. Various stability analysis tools, viz, root locus, step response, Bode plot were used to prove the closed-loop (CL) stability of these models. It was shown that both the FNPK and F-ROM models are CL stable for all combinations of τ and α. Further, it was shown with analysis that the linear of fractional ordermodels are more stable as compared to their integer-order counterparts (CNPK).

Realization of fractional-order operators (derivatives and integrals) and fractional-order models, both linear as well as nonlinearin general, is still an open problem. The fractional-order operators, being infinite dimensional in nature, are difficult to realize or simulate. Several continuous and discrete approximations like continued fraction expansion (CFE), Carlsons method, Matsudas method, Oustaloups method (Oustaloup et al., 2000), Charefs Method (Charef et al., 1992) in continuous time domain approximation and Eulers, Tustin, Al-Alaoui (Visweswaran et al., 2011), methods in discrete time approximation (Krishna, 2011) are available in literature. There are several numerical methods available to solve the linear and nonlinear fractional-order differential equations. Each of the approximation and numerical method has its own merits and demerits. In the mathematical theory of artificial neural networks(ANN) the universal approximation theorem states that a feed-forward network with a single hidden layer containing a finite number of neurons (that is, a multilayer perceptron), can approximate continuous functions on compact subsets of $\mathbb{R}^n$, under mild assumptions on the activation function (Zurada, 1992).

Neural networks can acquire, store and utilize experiential knowledge. The knowledge is in the form of stable states or mappings of embedded networks which can reproduce the same response to the presented data and even able to estimate the response for the input values which was not part of the training dataset. In view of this, the ANNs can be used to approximate the fractional-order operators and linear and nonlinear fractional-order systems. This paper, probably for the first time, presents the use of ANNs to approximate the linear fractional-order models (FNPK and F-ROM).

The use of ANNs to model nuclear reactor dynamics is a quite mature field. There is abundant literature available that report the ANN modeling of nuclear reactors. For example, a modified Back-propagation algorithm with adaptive steepness factor to speed up the training of RMLP is mentioned in Adali et al. (1997). Neural network has been imposed (Akkurt and Çolak, 2002) for the Estimation of system parameters of PWR system during transients. An approach to the on–line training of infinite impulse response-locally recurrent neural network (IIR-LRNN) has been modeled and compared with off–line batch mode training for the modeling the dynamics of Lead-Bismuth Eutectic-eXperimental Accelerator Driven System (LBE-XADS) nuclear reactor (Zio et al., 2009). The neural model has been designed and developed (Khalafi and Terman, 2009) for precise modeling of nuclear reactor to study the reactor core dynamics and predict various insertion rates. An adaptation of cellular neural network to model and solve nuclear reactor dynamics equation is employed in Hadad and Piroozmand (2007). The framework of locally recurrent neural networks for approximating the temporal evolution of a nonlinear dynamic system model of simplified nuclear reactor has been studied in Cadini et al. (2007).

This work employs the multilayer ANNs to approximate the FNPK and F-ROM models. The exercise requires the input-output data to train a network. For the FNPK model, the ANN approximations are obtained for time as single input and both time and derivative order α as two inputs. For F-ROM, the ANN approximations are obtained for only time as an input to the network. The MATLAB ANN toolbox is used for training the networks. It is shown with the help of simulations that the trained networks mimic the fractional-order models very closely and the error is very small. Also, in case of FNPK model, the proposed ANNs with input as α, are found to provide a satisfactory approximation for the intermediate values of fractional-order derivative.

The salient contributions of the paper are as follows:

1. Development of ANN models for FNPK models for reactor with unity feedback.
2. Development of ANN models for F-ROM models for reactor with unity feedback.
3. Analysis of the effect of fractional-order model structure on various ANN parameters, like number of layers, number of neutrons, time required for the training.
4. Analysis of the fractional-order of FNPK and F-ROM models and relaxation time constants on various ANN parameters.
5. Performance of the ANN model for intermediate values of fractional-order for FNPK.

The paper is organized as follows. Next section gives the fundamentals of fractional calculus and fractional-order systems in brief. Section 3 describes the linear models of fractional-order considered in this work for ANN modeling. The fundamentals of ANN modeling is given in Section 4. Section 5 presents the results and analysis. Conclusion is given in Section 6.

2. Fractional calculus and fractional-order systems

Fractional calculus is the field of mathematics dealing with the derivatives and integrals of non-integer, real or complex-order (Podlubny, 1999). The field, originated in 1695, has recently found applications in mathematical modeling, signal processing and control. The fractional derivative or integral operator is known as 'differentigral'. Fractional calculus and its applications are now quite mature fields. A rigorous literature survey and applications of fractional calculus are presented in (Ortigueira, 2011; Machado et al., 2011).

There are three popular definitions of fractional order derivative operator ${}^0D^\alpha \equiv \frac{d^\alpha}{dt^\alpha}$, Grünwald-Letnikov ($GL$) definition, Riemann-Liouville (RL) and Caputo (C) definitions (Podlubny, 1999). These are defined as follows. The GL derivative of fractional order $\alpha \in \mathrm{R}^+$, is given by

$$ {}^{GL}_{0}\mathrm{D}^\alpha f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{t/h} (-1)^j \, {}^{\alpha}C_j \, f(t - jh) \tag{1}$$

where f is a continuous function, t/h is the integer part, and ${}^{\alpha}C_j$ is the binomial coefficient.

Let $\alpha \in \mathbf{R}^+$, $n-1 < \alpha \leqslant n$, n integer, and f be a continuous function, then Riemann–Liouvillefrational-order derivative is defined as

$$ {}^{RL}_{0}\mathrm{D}^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \tag{2}$$

Caputo fractional-order derivative is defined as

$$ {}^{C}_{0}\mathrm{D}^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, \quad f(t) \in C^n \tag{3}$$