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Sensitivity and uncertainty analysis for the PWR online power-distribution monitoring with NECP-ONION system

Zhuo Li, Liangzhi Cao *, Hongchun Wu, Wei Shen, Yong Liu, Chenghui Wan

School of Nuclear Science and Technology, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China

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ABSTRACT

In this paper, the sensitivity and uncertainty analysis model has been proposed and implemented in the NECP-ONION system, which is capable of performing the 3D online power-distribution monitoring for PWR by applying the harmonics expansion method. A new method called the analytical method is used to quantify the relative sensitivity coefficients of the monitored power distributions with respect to the detector signals; then the "sandwich" rule is used to quantify the uncertainties of the monitored power distributions with the sensitivity-analysis results based on the uncertainty-propagation method. The BEAVRS benchmark is used to verify the sensitivity and uncertainty analysis capability in NECP-ONION. The results of the direct numerical perturbation method are used as reference to verify the sensitivity coefficients calculated with the analytical method. The results of the statistical sampling method and the analytical method are compared to each other for the uncertainty verification. Then, the sensitivity and uncertainty analyses with various expansion orders of harmonics and core burnup levels are carried out. Numerical results show that, the expansion order is the trade-off of the monitoring accuracy and uncertainty, while the sensitivity and uncertainty analysis is useful for the determination of the expansion order.

1. Introduction

The 3D online power-distribution monitoring is important on account of its impact to the safety and the economy of a plant. It can be employed to determine the power peaking factor, the enthalpy rising factor, the quadrant tilt ratio, and the fuel burnup distribution. The calculation accuracy of these parameters affects the operation limits and the safety margins of a reactor. Besides, online power-distribution monitoring can be employed to reduce the over-conservative operating principles and hence improve the economy of nuclear power plants. Therefore, most commercial power reactors in operation or under-built are equipped with a 3D online power distribution monitoring system.

Actually, this subject has been studied from 70s of last century. Several representative online monitoring systems have been developed by using in-core or ex-core detectors, such as RFSP flux mapping for CANDU (Hinchley and Kugler, 1974; Shen and Schwanke, 2012), BEACON (Beard and Morita, 1988) for PWR, SCORPIPO-VVER (Pecka et al., 1999) for VVER and ABB CORE MASTER (Lundberg and Wenisch, 1999) for BWR. In recent years, as the commencement of construction for a new generation reactor core, the 3D online power-distribution monitoring attracts more and more attention, aiming to obtain the power distribution in real time and guarantee the operation state of a reactor core. In this context, many computational methods have been proposed, including harmonics expansion method (HEM) (Wang et al., 2011), thin-plate spline method (Boyd and Miller, 1996.), internal boundary condition method (Chan and Mamourian, 1990), coupling coefficients method (Terney et al., 1983), least-square method (Lee and Kim, 2003), error shape synthesis method (Hong, 2004), weight coefficient method, polynomial expand method (Li et al., 2013) and ordinary kriging method (Peng et al., 2014), etc.

In the past, research for the online power-distribution monitoring mainly focused on the monitoring method. How to combine and solve the neutron diffusion equation with the detector signals has been investigated by some studies. However, the sensitivity and uncertainty (S&U) analysis for the results, which is as important as the monitoring method, is mostly ignored. Monitoring results without uncertainty are published based on several monitoring methods. However, they are not completely from the safety-evaluation principle of "best estimate plus uncertainty analysis". The monitored power distribution will affect the operation limits and the safety margins of a reactor. Therefore, it is demanded to provide the confidence interval of the monitored

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* Corresponding author.
E-mail address: caolz@mail.xjtu.edu.cn (L. Cao).

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power distribution in the online power-distribution monitoring. In this context, S&U analysis is necessary for the monitoring system to obtain uncertainty to provide more information of the safety margin.

For the S&U analysis, two types of methods have been widely used, the deterministic-based method and the statistical sampling method. In the deterministic-based method, the sensitivity analysis is carried out first with either the direct numerical perturbation (DNP) method or the perturbation-theory (PT) method (Liu et al., 2015) to obtain the sensitivity coefficients of responses. The "sandwich" rule is used subsequently to calculate the uncertainties of responses by combining the sensitivity coefficients with the covariance matrix of input parameters. In the statistical sampling method, samples are firstly generated from the distribution regions of inputs according to the covariance matrix of input-parameter uncertainties. Repetitive calculations are performed subsequently with various samples as inputs. Then, the statistical calculation is adopted to obtain the uncertainties of responses. Compared with the deterministic-based method, the statistical sampling method has advantages of no low-order approximation, no limit to the type and number of responses (Wan et al., 2015), and convenient for the uncertainty analysis for the responses without explicit equations expressions. But on the other hand, the statistical sampling method has the disadvantage of large calculation cost and cannot obtain the sensitivity coefficients which are important for the uncertainty source analysis, similarity analysis and nuclear data assimilation (Broadhead et al., 2004).

In our previous work (Li et al., 2017), the NECP-ONION system has been developed and verified, which has the capability to perform the 3D online power-distribution monitoring for PWR, by applying the harmonics expansion method as the major monitoring method. In this paper, the S&U analysis has been proposed and implemented in NECP-ONION, which makes the system integrated including plant data processing, core calculation, online power-distribution monitoring and S&U analysis. Considering the requirement of the computational speed for online monitoring, the deterministic-based method is selected to perform the S&U analysis. But different from the DNP or PT method, a new method called the analytical method is applied for the sensitivity coefficient calculation, whose application is owing to the explicit equations expressions of harmonics expansion method. The "sandwich" rule is used to quantify the uncertainties of the monitored power distributions with the results of sensitivity analysis based on the uncertainty-propagation method. The BEAVRS benchmark (Horelik et al., 2017) is used to verify the S&U analysis capability in NECP-ONION. The results of the DNP method are used as reference to verify the sensitivity coefficients calculated with the analytical method. The results of the statistical sampling method and the analytical method are compared to each other for the uncertainty verification. Then, the S&U analyses with various expansion orders of harmonics and core burnup levels are carried out.

The remainder of this paper is organized as follows. Section 2 describes theoretical models of the harmonics expansion method for the online monitoring and the analytical method used for the S&U analysis. The verification results are given in Section 3. Section 4 describes the application of S&U analysis with various expansion orders and core burnup levels. Finally, summary and conclusions are presented in Section 5.

2. Method

2.1. Harmonics expansion method

Harmonics have exact definition which is based on the neutron-diffusion equation.

$$\boldsymbol{M\Phi} = \frac{1}{k}\boldsymbol{F\Phi} \tag{1}$$

where $\boldsymbol{M}$ and $\boldsymbol{F}$ are the destruction and production of neutrons respectively, $\boldsymbol{\Phi}$ is the neutron flux and k is the effective multiplication factor.

Then Eq. (1) can be written as follow:

$$\boldsymbol{A\Phi} = k\boldsymbol{\Phi} \tag{2}$$

where $\boldsymbol{A} = \boldsymbol{M}^{-1}\boldsymbol{F}$.

This equation is an eigenvalue equation and it has a series of eigenvalues $\{k_1, k_2, \ldots, k_n, \ldots\}$ and the corresponding eigenvectors $\{\boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2, \ldots, \boldsymbol{\Phi}_n, \ldots\}$. These eigenvectors are called harmonics. It has been proved that harmonics have properties of completeness and orthogonality. Then harmonics compose a set of complete basis functions.

The harmonics expansion method has explicit equations. Taking the two-energy-group approximation as an example, reactor power distribution can be expanded by harmonics:

$$P(r) = \sum_{n=1}^{N} a_n\left(\kappa\Sigma_{f1}(r)\Phi_{1,n}(r) + \kappa\Sigma_{f2}(r)\Phi_{2,n}(r)\right) \tag{3}$$

where $P(r)$ is the expanded power distribution, r is the spatial variable, $\kappa\Sigma_{f1}$ and $\kappa\Sigma_{f2}$ are the energy production cross sections, N is the expansion order and a_n is the nth-order expansion coefficient. The expansion order N is set to 30 as default in this paper.

Detector signals can be written as Eq. (4).

$$s(r_d) = C_d N_d \sum_{n=1}^{N} a_n(\sigma_1(r_d)\Phi_{1,n}(r_d) + \sigma_2(r_d)\Phi_{2,n}(r_d)) \tag{4}$$

where $s(r_d)$ is the detector signal at r_d, C_d is the proportion coefficient of detector at r_d, N_d is the detector-material nucleus-density at r_d, σ_1 and σ_2 are the two-group micro cross-sections of the detector material.

It should be noticed that the proportion coefficient C_d is a property of the detector itself and is mostly affected by the geometry and material of the detector. In practice, it will also be influenced by the electrical elements such as the signal-amplifier and cable. Therefore, this is a parameter determined by the manufacture factory and calibrated by the reactor measurements. In this paper, this proportion coefficient is set to a constant for an online power-distribution monitoring calculation.

The detector response $R(r_d)$ is defined as follow:

$$R(r_d) = \frac{s(r_d)}{C_d N_d} \tag{5}$$

If the number of detectors is M, equations can be obtained as follow:

$$\begin{cases} R(r_1) = \sum_{n=1}^{N} a_n(\sigma_1(r_1)\Phi_{1,n}(r_1) + \sigma_2(r_1)\Phi_{2,n}(r_1)) \\ R(r_2) = \sum_{n=1}^{N} a_n(\sigma_1(r_2)\Phi_{1,n}(r_2) + \sigma_2(r_2)\Phi_{2,n}(r_2)) \\ \ldots \\ R(r_M) = \sum_{n=1}^{N} a_n(\sigma_1(r_M)\Phi_{1,n}(r_M) + \sigma_2(r_M)\Phi_{2,n}(r_M)) \end{cases} \tag{6}$$

Then, the expansion coefficients can be solved via the least-square principle. The monitored power distribution can be obtained by Eq. (3) subsequently.

2.2. Analytical method for the sensitivity coefficient calculation

As mentioned above, the explicit equations of harmonics expansion method makes the usable of the analytical method. Different