Reliability Engineering and System Safety 147 (2016) 109–116

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Kernel estimator of maintenance optimization model for a stochastically degrading system under different operating environments

I.B. Sidibé [a,*], A. Khatab [b], C. Diallo [c], K.H. Adjallah [d]

[a] Laboratory of Industrial Engineering, Production and Maintenance (LGIPM), University of Lorraine, Metz, France
[b] Laboratory of Industrial Engineering, Production and Maintenance (LGIPM), National School of Engineering, Metz, France
[c] Dalhousie University, Halifax, Canada
[d] Laboratory of Systems Design, Optimization and modelling (LCOMS), National School of Engineering, Metz, France

ARTICLE INFO

Article history:
Received 26 September 2014
Received in revised form
15 September 2015
Accepted 8 November 2015
Available online 19 November 2015

Keywords:
Nonparametric estimation
Kernel estimation
Maintenance optimization
Operating environment

ABSTRACT

This paper investigates the preventive age replacement policy (ARP) for a system subject to random failures. Unlike most maintenance models in the literature, our model considers a system that is exploited under different operating environments each characterized by its own degree of severity. The system lifetimes follow a different distribution depending on the environment it is operating under. Furthermore, the system lifetimes distribution is assumed unknown and therefore estimated from field reliability data. The reliability of the system is calculated using two kernel estimators. This method offers the advantage of non-parametric estimation methods and completely determined by two parameters, namely the smoothing parameter and the kernel function. First, a probability maintenance cost model is derived and conditions under which an optimal preventive maintenance age exists are provided. Then, a statistical maintenance cost model is developed using two kernel estimators. The impact of the variability of the kernel smoothing parameter on the cost model is also investigated. Numerical experiments are provided to illustrate the proposed approach. Results obtained demonstrate the accuracy of the proposed statistical maintenance cost model.

1. Introduction

To reduce the risk of production systems failures and deterioration, maintenance activities should be performed according to an appropriate schedule. The growing importance of maintenance activities for stochastically degrading production systems has led to an increasing interest in the development and implementation of maintenance optimization models. Many interesting and significant results for a huge variety of maintenance optimization models have been developed in the literature. Early works have appeared in [1] and [2]. Subsequently, many extensions of maintenance mathematical models appeared in the literature. For a survey, the reader may refer, for example to [3–6], and the references therein. In [7] and [8], the authors proposed maintenance models dealing with finite time horizon.

Most existing maintenance models merely rely on the classical assumption that the distribution of the system lifetimes is well characterized and a priori known. Furthermore, the majority of existing maintenance optimization models assume that the environment where the system is being exploited is static and thus does not induce any changes to the system reliability characteristics, i.e. the degradation process is the same during the system's life cycle. This is a restrictive assumption as in many industrial areas, production and service systems may experience different operating environments each characterized by a degree of severity that impacts their performance. For example, the degradation process of a mining machinery is impacted by the severity level of the environment conditions under which the machinery is being exploited: mud, dust, humidity, etc. Another example could be the engines used for oil extraction. The degradation process of such systems depends on whether they are operated onshore or offshore. In some other industries, production systems are first operated in a given environment and then moved to another location where the operating environment may be more or less severe than the first. Many companies operate their machinery in their home countries for many years before shipping them to their subsidiaries in other countries where they will be subjected to more severe operating conditions. Therefore, efficient maintenance strategies integrating the heterogeneous operating

* Corresponding author.
E-mail addresses: bouransidibe@gmail.com (I.B. Sidibé), khatab@enim.fr (A. Khatab), Claver.Diallo@dal.ca (C. Diallo), adjallah@enim.fr (K.H. Adjallah).

http://dx.doi.org/10.1016/j.ress.2015.11.001
0951-8320/

conditions should be developed to assess the degradation of such systems and propose appropriate maintenance strategies.

In this paper, a preventive maintenance (PM) policy is investigated for a system subjected to random failures. The system is assumed to have previously operated under a given environment and is now exploited under a second environment. Each operating environment is characterized by its own degree of severity which in turn impacts the system's reliability. To reduce the risk of failure, while being exploited in the second operating environment, the system undergoes the ARP according to which the system is replaced either at failure (corrective replacement) or at a given age T (preventive replacement); whichever occurs first. The objective is to find the optimal replacement age T which minimizes the expected total maintenance cost per unit of time. This total expected cost is the sum of the corrective and preventive replacement costs. From field data recorded from the two operating environments, a statistical maintenance cost model is derived using the nonparametric kernel method. Such an estimation method is completely defined by the definition of two elements: the smoothing parameter and the kernel function. The robustness of the proposed statistical maintenance model is demonstrated by showing that only the smoothing parameter variability may impact the total maintenance cost. Furthermore, under some very mild conditions, related to the data sample size and the elements of kernel estimator, the maintenance cost is shown to remain unchanged even when the kernel smoothing parameter varies.

Optimization of maintenance problems using the kernel estimation method has, to the best of our knowledge, only been proposed by Coolen and his co-authors in [9] and [10]. In these works, the ARP is also investigated and a statistical maintenance cost model is provided using a discrete distribution estimator. The optimal replacement age is determined by minimizing the upper and lower bounds of the average total maintenance cost per unit of time. In another work [11], the authors used the kernel estimation method to model and optimize a software system availability problem.

Unlike what Coolen and Coolen proposed in [10], our approach, is completely based on the kernel estimation of a continuous cost model corresponding to the age replacement policy. Such a model avoids computing cost function bounds but rather allows the computation of exact optimal age of preventive replacement. Moreover, a sensitivity analysis is carried out to evaluate the effects of the kernel parameters on the estimated age of replacement. In addition, the present work investigates the age replacement policy for systems assumed to accomplish their missions under different operating environments.

The rest of this paper is organized as follows. In Section 2, the maintenance problem is described and a probability maintenance optimization model is then proposed. Conditions under which the optimal age replacement exists are also addressed in Section 2. On the basis of kernel estimation method, a statistical maintenance cost model is proposed in Section 3. In Section 4, numerical experiments are conducted to illustrate the proposed approach. The results obtained will serve to demonstrate the accuracy and robustness of the statistical maintenance model. Conclusions and future works are drawn in Section 5.

2. Probability maintenance optimization cost model

In this work, we consider a system that has previously functioned under an operating environment (OE_1) and at time $t=T_1$ has started to work under a new operating environment (OE_2) which is assumed to be more severe than OE_1. The system failure process being more intensive in OE_2 than in OE_1, the same level of system degradation is accumulated more rapidly in OE_2 than in

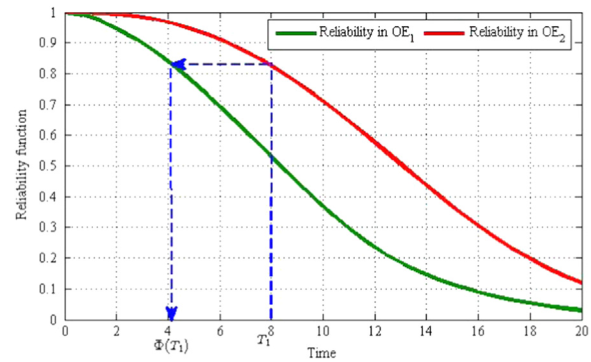

Fig. 1. Reliability function for a system under two operating environments.

OE_1. It follows that operating for T_1 time units in OE_1 is equivalent to operating for $T_2 < T_1$ time units in OE_2. A correspondence relationship can be derived to obtain the equivalent age of the system when shifting between operating environments. Let $R_j(t)$, $F_j(t)$ and $\lambda_j(t)$ denote, respectively, the reliability function, the cumulative distribution function and the failure rate of the system when it operates in OE_j $(j=1,2)$. The correspondence relationship (a function), hereafter denoted by $\Phi(t)$, is based on the concept of *statistical virtual age* initially introduced in the work by Finkelstein [12,13]. According to this concept, $\Phi(T_1)$ is the *virtual* age of the system immediately after switching from OE_1 to OE_2. This virtual age satisfies

$$R_1(T_1)=R_2(\Phi(T_1)) \quad \text{and} \quad \Phi(0)=0. \tag{1}$$

The above equations are derived while assuming that the system is more reliable in OE_1 than in OE_2, i.e. $R_1(t)>R_2(t)$ for $t\geq 0$ which in turn states that lifetimes of the system in the two OE are stochastically ordered. An example is given in Fig. 1 where reliability functions of the system in OE_1 and OE_2, respectively, $R_1(t)$ and $R_2(t)$ are drawn with respect to time. It can be noted that $R_1(t)>R_2(t)$ which states that in OE_2, the system is less reliable than in OE_1. It follows that operating T_1 units of time in OE_1 corresponds to operating for $\Phi(T_1)$ units of time in the second environment with $\Phi(T_1)<T_1$. For illustration, assume that the system accomplished its mission in OE_1 with an age of $T_1=8$ units of time and is then operated under OE_2. The virtual age $\Phi(T_1)$ corresponding to the initial age T_1 immediately after the change of environment is determined to be $\Phi(T_1)=4$. Accumulating the same level of system degradation requires 8 units of time under OE_1 while its requires only half under OE_2.

The transfer function $\Phi(t)$, developed to compute the virtual age of system after environment changing, is non-decreasing and considered unknown and rather estimated from failure data. These data are collected from both operating environments OE_1 and OE_2. Therefore the reliability distribution functions are firstly estimated and the transfer function is deduced for each initial age T_1 of the system.

To reduce the system downtime and to achieve higher performance, the system will be subjected to PM according to the ARP. Costs of corrective and preventive replacements are denoted, respectively, by C_r and C_p (with $C_p \ll C_r$). The objective is then to determine the optimal replacement age T_0 which minimizes the expected total maintenance cost per unit of time $C_{T_1}(T)$ incurred by the corrective and preventive replacements.

The probability model corresponding to the total expected maintenance cost per unit time $C_{T_1}(T)$ is

$$C_{T_1}(T)=\frac{E[\text{Total maintenance costs per cycle}]}{\text{Average cycle length}},$$

$$C_{T_1}(T)=\frac{C_r\times F_2(T+\Phi(T_1)|_{\Phi(T_1)})+C_p\times R_2(T+\Phi(T_1)|_{\Phi(T_1)})}{\int_0^T R_2(t+\Phi(T_1)|_{\Phi(T_1)})\,dt}, \tag{2}$$