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A Bayesian negotiation model for quality and price in a multi-consumer context

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ABSTRACT

Bayesian decision theory plays a significant role in a large number of applications that have as main aim decision making. At the same time, negotiation is a process of making joint decisions that has one of its main foundations in decision theory. In this context, an important issue involved in industrial and commercial applications is product reliability/quality demonstration. The goal is, among others, product commercialization with the best possible price. This paper provides a Bayesian sequential negotiation model in the context of sale of a product based on two characteristics: product price and reliability/quality testing. The model assumes several parties, a manufacturer and different consumers, who could be considered adversaries. In addition, a general setting for which the manufacturer offers a product batch to the consumers is taken. Both the manufacturer and the consumers have to use their prior beliefs as well as their preferences. Sometimes, the model will require to update the previous beliefs. This can be made through the corresponding posterior distribution. Anyway, the main aim is that at least one consumer accepts the product batch based on either product price or product price and reliability/quality. The general model is solved from the manufacturer viewpoint. Thus a general approach that allows us to calculate an optimal price and sample size for testing is provided. Finally, two applications show how the proposed technique can be applied in practice.

1. Introduction

The problem of reliability/quality demonstration of products or a system has gained large interest in recent years, due to the increasing industrial demands (see [2]). At the same time, reliability demonstration of a system can be viewed as a Bayesian decision problem in which the acceptance of the system with respect to its reliability is analyzed (see [25]). Thus, the principles of Bayesian decision theory can be applied (see [3]). Rufo et al. [31] presented a Bayesian decision making approach for the sale of a product batch in the context of life testing. Specifically, a Bayesian sequential negotiation model for which a manufacturer offers a product batch to a consumer is considered. The decision of accepting or rejecting the product batch is based on the reliability of the product, which is represented by its lifetime.

Negotiation can be defined as a process of joint decision making (see [39,35]). That is, a process by which two or more parties try to reach compromises and to come to an agreement. Negotiations occur in all domains of life (see, e.g., [37]). However, the basic structure of negotiations applied to different contexts is basically the same (see [23]). Thus, any negotiation process has the following common characteristics (see [26]): (i) there are two or more parties involved in the entire procedure; (ii) the payoffs for each party depend either on the consequences of the joint decisions or on alternatives external to the negotiations; (iii) the parties can reciprocally and directly exchange information (this communication can be honest or not); (iv) the parties can be creative in the decisions they make in order to arrive at a joint decision. Current reviews on negotiation analysis are provided by Tsay and Bazerman [38] and Agarwal [1].

Zheng et al. [41] pointed out that the existing literature about mathematical study of negotiation can be divided into two classes: mediated and non-mediated negotiation. In the first case, the parties interact with each other through a mediator. In non-mediated negotiation, the parties communicate with one another directly. By considering the literature for this last type of negotiation, different suppositions can be made. They are related to the number of parties, i.e., bilateral or multilateral negotiation (see, e.g., [22]) and the number of issues to be negotiated, i.e., single issue or multi-issue negotiation (see, for instance, [8]). Most literature focuses on negotiation model by considering two parties and one issue (see, e.g., [40]) or two parties and several issues (see [9,6,12,42], among others). However, nowadays there exists

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literature dealing with several parties and issues. Moreover, it applies negotiation to different fields (see, for instance, [21,28,27]). Other applications that consider negotiation teams are provided by Sánchez-Anguix et al. [33,34].

A difficulty when dealing with both single issue and multi-issue negotiation is to characterize the preferences of a party through the utility function. Therefore, the parties will make decisions based on the corresponding utility function. In addition, another matter to take into account is to assume whether parties that negotiate have complete or incomplete information about their opponents' preferences (utility functions). Observe that, in the last case, it is harder to come to an optimal agreement. Lai et al. [16] presented a more detailed review.

In a wide variety of industrial and commercial applications, the product reliability has an influence on the parties' preferences. Much of the literature on reliability and survival analysis deals with the topics of life testing and the analysis of failure data. Furthermore, in most cases involving industrial settings, life-testing is performed with the aim of making decisions (see, e.g., [36]). Lindley and Singpurwalla [18,19] proposed a Bayesian framework which involved a single issue and two adversarial decision makers: the manufacturer and the consumer. The scenario in which the consumer demands a batch of items from the manufacturer for testing is considered. The consumer could either accept or reject the batch provided by the manufacturer. This decision is based on a single issue that is the product reliability. If the consumer chooses to reject the batch, then the two main decisions brought up are the following: (i) whether the manufacturer should offer a sample to the consumer for testing, and (ii) how large it should be. Lindley and Singpurwalla [18] discuss the problem above in the context of acceptance sampling for quality control by using Bernoulli, Poisson and Normal distributions, whereas Lindley and Singpurwalla [19] consider exponential lifetimes in a reliability context.

This paper presents a Bayesian sequential negotiation model among multiple parties (a manufacturer and several consumers) on two issues: price and product reliability/quality. Thus, the procedure presented in Rufo et al. [31] is extended and modified by considering different consumers and by introducing the product price as well as its quality as decision criteria. The mediator's presence is not required. Thus, the manufacturer directly interacts with the consumers. In addition, it is considered that each consumer does not have knowledge about the preferences and judgements of the remaining consumers. The decision problem is solved under the manufacturer viewpoint. Firstly, it is assumed that the manufacturer offers the consumers a product batch by considering an initial price. Then, the consumers can choose between two alternatives that consist of accepting or rejecting the product batch. This decision is made based on the consumers' preferences and beliefs. If the second alternative is selected, then the manufacturer considers the possibility of convincing at least one of the consumers that the manufacturer's perspective is more correct. In order to do it, she/he has two options: to modify the initial price and/or to show that the product reliability/quality is high, providing the consumers with a sample.

A Bayesian approach is implemented to obtain both the price and the optimal sample size that the manufacturer will offer the consumer. The model is computationally complex since it involves the computation of some distributions together with some expected utilities. Thus, a direct implementation is possible when analytical expression can be obtained. In other cases, efficient Monte Carlo methods will be used. Finally, it is shown through two applications that the proposed approach can be used in several decision making settings.

The outline of the paper is as follows. In Section 2, the general model is described together with a suitable negotiation protocol. Moreover, a straightforward approach is proposed in order to obtain both an appropriate price and an optimal sample size. Section 3 applies the developed methodology to several distributions. Firstly, a set of distributions coming from the exponential family is taken. Hence, a unified framework is provided. That is applied to the Weibull case. For the second application, Bernoulli sampling is proposed. Finally, conclusions are presented in Section 4.

2. A general negotiation framework

Suppose that several parties (one manufacturer and different consumers) are negotiating the sale of a product. The main criteria for this sale are based on the price and the reliability/quality of the product. Both manufacturer and consumers are involved in a negotiation process in which they are expected to come to an agreement. In addition, it is assumed that the sale is made so that only one consumer is buying all products. On the other hand, the manufacturer is the only party who has additional information (preferences and judgements) about the remaining parties (consumers) throughout the negotiation process.

Firstly, the manufacturer offers m (>1) consumers a product bath composed of K units with an initial price z_0. Note that when $m=1$, a consumer is the only party negotiating with the manufacturer. The product has a characteristic under study which is denoted by the random variable X. Thus, for the product batch and given a parameter θ, it is considered that the random variables, X_i, $i=1,2,\ldots,K$ are independent and identically distributed with distribution depending on the parameter θ. Observe that this parameter is directly related to product characteristics such as its reliability/quality.

From a consumer's viewpoint, $\mathcal{C}_j$, $j=1,2,\ldots,m$, she/he might accept or reject the K units of the product based on her/his prior knowledge about θ, $\pi_{\mathcal{C}_j}(\theta)$, and the utility function, $\mathcal{U}_{\mathcal{C}_j}$, which represents the preferences for $\mathcal{C}_j$. In this paper, the beliefs of both the manufacturer and the consumers are elicited as conjugate prior distributions. This choice has two main advantages. Firstly, the posterior distributions have the same functional form as the prior distributions. Secondly, analytical expressions for the predictive prior distributions involved in the process can be easily obtained. On the other hand, for the utility functions, the preference criteria are mainly based on a trade-off between the product reliability/quality and the price paid for the product batch. Moreover, two alternatives are considered: accepting or rejecting this batch. Specifically, the expressions for the utilities of accepting are composed of two parts. The first one, $A_{1_j}(\theta)$, is referred to the reliability/quality. The second part, $A_{2_j}(z)$, includes the remaining criteria for accepting the product batch, i.e., price, which is denoted by z, brand equity, availability to find an agreement, etc. Hence, the utilities of accepting the product batch are,

$$\mathcal{U}_{\mathcal{C}_j}(A,\theta,z)=A_{1_j}(\theta)+A_{2_j}(z), \quad \text{for } j=1,2,\ldots,m, \tag{1}$$

whereas the utility function of rejecting is given by:

$$\mathcal{U}_{\mathcal{C}_j}(R,\theta,z)=a_{3_j}, \quad \text{for } j=1,2,\ldots,m,$$

where $a_{3_j}\in\mathbb{R}$ is a constant, representing whether the consumer is willing to reach an agreement. Observe that the utilities of accepting are additive functions. Then, the consumer, $\mathcal{C}_j$, will accept the product batch when it is satisfied that the expected utility of accepting, is greater than or equal to the expected utility of rejecting, that is:

$$E_{\pi_{\mathcal{C}_j}(\theta)}\left[\mathcal{U}_{\mathcal{C}_j}(A,\theta,z)\right]=\int_{\Theta}\mathcal{U}_{\mathcal{C}_j}(A,\theta,z)\pi_{\mathcal{C}_j}(\theta)\,d\theta \geq \int_{\Theta}\mathcal{U}_{\mathcal{C}_j}(R,\theta,z)\pi_{\mathcal{C}_j}(\theta)\,d\theta=E_{\pi_{\mathcal{C}_j}(\theta)}\left[\mathcal{U}_{\mathcal{C}_j}(R,\theta,z)\right]. \tag{2}$$