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Technical note

On the equivalence of neutron source and flux spectra

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ABSTRACT

The equivalence between the neutron source and flux spectra is often implicitly used in practice although many times the users are not even aware of this. This work identifies two conditions under which the equivalence holds. Namely, if the neutron interaction between the source region and the volume where flux is observed is negligible, and the neutron mean track length in the observed volume does not depend on their energy, source and flux spectra are equivalent. Consequently, a flux determined on a closed surface from a full system calculation can be replaced by an equivalent source for a simplified model including only the region contained by the surface.

1. Introduction

The equivalence between the neutron source and flux spectra is often implicitly used in practice although many times the users are not aware of the conditions under which this equivalence holds. One example of application is a pre-calculated neutron flux spectrum for definition of a source spectrum in simplified-geometry model for reaction rate calculations (Trkov et al., 2009; Žerovnik et al., 2015; ICSBEP, 2015).

For an arbitrary time-independent neutron transport problem in vacuum, the ratio between the number of source neutrons and any surface or volume averaged neutron flux is independent of neutron energy as long as the spatial and angular distribution of the source neutrons remains unchanged. This is obvious for this particular case (i.e. vacuum) since only geometric parameters define the flux/source ratio. When adding material which interacts with neutrons, energy dependence is introduced in the ratio through energy-dependent probabilities for neutron-nucleus interactions (i.e. energy-dependent neutron induced cross sections).

Furthermore, the time-independent neutron transport in a medium is affected by probabilities for interaction of the neutrons with the surrounding material (reaction cross section) and the system geometry, and not directly on neutron energy (or speed), except for energy-dependent reaction cross sections. Since the volume-averaged neutron flux is proportional to the track length in the volume (so-called track length estimators used in Monte Carlo codes such as MCNP (Goorley et al., 2012)), the neutron flux in an arbitrary energy bin also depends only on the number of neutrons passing the volume, the incoming position and angle, and macroscopic cross section in the volume in case the mean chord length of the volume is not negligible compared to the mean neutron free path in the material. It again does not depend directly on neutron energy/speed but indirectly due to energy-dependent reaction cross sections.

As rigorously shown below, the neutron source spectrum is thus equivalent to the neutron flux spectrum and not to the neutron density spectrum. Neutron density in any point/volume in the system for a monoenergetic source and vacuum or energy independent cross section is inversely proportional to the neutron speed.

Important implication of this finding is that one may directly use the results of a calculated neutron flux for definition of a source in the same geometry to propagate neutrons and save computational time. This work looks at the underlying principles of the flux-source equivalence and tries to determine the conditions that need to be satisfied for this equivalence to hold.

2. Definitions

Angular neutron density (Duderstadt and Hamilton, 1976):

$$n(\vec{r}, \vec{\Omega}, E, t), \tag{1}$$

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where $\vec{r}$ is the position of the observed neutron in space, $\vec{\Omega}$ its direction, E its energy and t the time.

Angular neutron flux (Duderstadt and Hamilton, 1976):

$$\Phi(\vec{r},\vec{\Omega},E,t) \equiv v(E)n(\vec{r},\vec{\Omega},E,t), \tag{2}$$

where $v(E)$ is the neutron speed.

Neutron source spectrum can be expressed as:

$$N(E) = S_0 v_s(E), \tag{3}$$

where the S_0 is the neutron source emission rate of the source and $v_s(E)\mathrm{d}E$ is the fraction of neutrons emitted with energy between E and $E + \mathrm{d}E$ (so that $\int v_s(E)\mathrm{d}E = 1$).

3. Source and flux spectrum equivalence

Let us imagine a monoenergetic neutron source with emission rate $N(E_0)$ with speed $v = \sqrt{2E_0/m_\mathrm{N}}$ located in a volume V.

In a medium with negligible scattering the average neutron density is:

$$n = \frac{Npt_\mathrm{p}}{V}, \tag{4}$$

where t_p measures the neutron's average time-of-flight through the selected volume, and p is the fraction of source neutrons entering the volume V. This time is connected to the average track length in the selected volume l_tr and speed v as:

$$t_\mathrm{p} = \frac{l_\mathrm{tr}}{v}. \tag{5}$$

This means that the neutron density is not proportional only to the source rate N at given energy E_0 but also inversely proportional to the neutron speed (or energy):

$$n = \frac{Npl_\mathrm{tr}}{vV}, \tag{6}$$

On the other hand, the neutron flux by definition equals to:

$$\Phi = N\frac{pl_\mathrm{tr}}{V}. \tag{7}$$

If l_tr and p are energy independent, the flux spectrum is proportional to the source spectrum. This is also demonstrated in Fig. 1.

The assumption of energy independent p is valid for a system where the neutron transport from the source to the volume V is not affected by neutron energy. In practice, this means that the source has to be located within the volume V or optically close to it and there is no significant room return. The assumption of energy independent l_tr is valid if the neutron transport in the volume V is not affected by neutron energy. This is always true for any surface or optically thin volume. It follows that for any neutron transport system, this equivalence can be used to simplify/accelerate the neutron transport calculation in an enclosed region of interest. A full scale model of the neutron transport system can be used to calculate neutron flux on a surface (or thin volume) enclosing the region of interest. The incoming (entering the region of interest) neutron flux can directly be used for definition of a neutron source on the surface of the simplified model of the system including only the region of interest thereby accelerating the calculation procedure.

3.1. Example: Reaction rates in ^{252}Cf spontaneous fission spectrum

The ^{252}Cf is used as a reference neutron source and its spontaneous fission spectrum is well characterized. It can be approximated with a Maxwellian spectrum (Snoj et al., 2012):

$$N(E) = C\sqrt{E\exp\left(-\frac{E}{a}\right)}, \tag{8}$$

where C is normalization constant and $a = 1.42$ MeV is model parameter. Alternatively, one could use the spectrum calculated by a sophisticated nuclear model, like the Madland–Nix model (Madland and Nix, 1983), or use an evaluated neutron spectrum, e.g. from the IRDF-2002 nuclear data library (IAEA, 2003).

The reaction rate of neutrons with a target can be described as:

$$R(\vec{r}) = \int_E \Sigma(\vec{r},E)\Phi(\vec{r},E)\mathrm{d}E, \tag{9}$$

where the Σ is the macroscopic cross section for interaction with neutrons. Usually, the neutron flux spectrum needs to be calculated at the target, before the integral can be evaluated. However, as shown in Section 3, in some cases the neutron source spectrum can be directly used instead of the flux. This makes calculations for reference sources, like the ^{252}Cf, more convenient, since only normalization needs to be determined. In such simple cases, computationally expensive Monte Carlo simulation is not needed.

4. Flux/source equivalence in Monte Carlo transport simulations

In Monte Carlo particle transport codes such as MCNP, the total average flux over a volume V_j is calculated as:

$$\phi_j = \int_{V_j}\frac{\mathrm{d}V}{V_j}\int_E \mathrm{d}E\int_{4\pi}\mathrm{d}\Omega\int_t \mathrm{d}t\Phi(\vec{r},\vec{\Omega},E,t). \tag{10}$$

In most applications, the system is in a steady state, thus the integration may be performed over the entire time interval. Under this assumption, the quantity ϕ_j is proportional to the neutron flux in a cell j corresponding to the volume V_j. The ϕ_j corresponds to the physical quantity of fluence, i.e. time integrated flux, by definition.

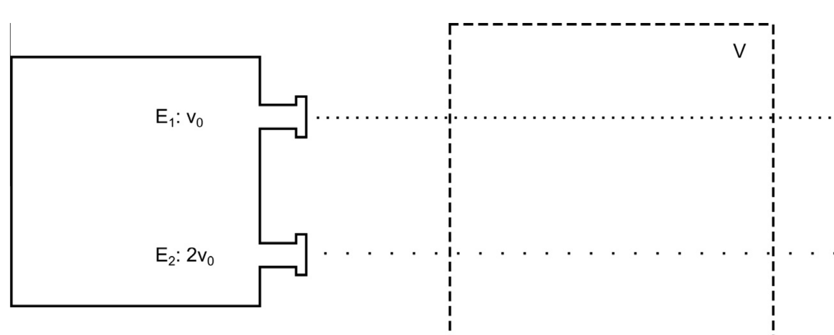

Fig. 1. The neutron source emits neutrons with two energies at the same rate. The density of the slow neutrons in the volume V is higher than that of the fast neutrons. The flux, however, is the same for both.