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Spectral properties of dynamic processes in a nuclear reactor

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ABSTRACT

As a rule, mathematical modeling of dynamic processes in nuclear reactors is conducted using an approach that treats a neutron flux in the multigroup diffusion approximation. In this approach, the basic model involves a multidimensional system of coupled parabolic-type equations. Similarly to common thermal phenomena, it is possible here to separate a regular mode of nuclear reactor operation that is associated with a selfsimilar development of a neutron field at large times. In this case, the main feature of dynamic processes is a minimal eigenvalue of the corresponding spectral problem. In the present paper, calculations of various eigenvalues are performed via the two-group model and discussed for the VVER-1000 reactor without a reflector and HWR reactor.

1. Introduction

The physical processes in a nuclear reactor (Duderstadt and Hamilton, 1976) depend on distribution of neutron flux, whose mathematical description is based on the neutron-transport equation (Hetrick, 1971; Stacey, 2007). The general view of this equation is integro-differential one, and the required distribution of neutrons flux depends on time, energy, spatial and angular variables. As a rule, the simplified forms of the neutron transport equation are used for practical calculations of nuclear reactors. The equation system that is known as a multigroup diffusion approach is mostly used for reactor analysis (Marchuk and Lebedev, 1986; Lewis and Miller, 1993; Sutton and Aviles, 1996; Cho, 2005) and is applied in most engineering calculation codes.

Modern reactor simulations are actually based on transport calculations (see, for example, Smith and Rhodes, 2002; Sanchez, 2012; Boyd et al., 2014). In multiscale reactor-physics simulations diffusion models are derived and applied using sophisticated homogenization methodologies Sanchez (2009) which define parameters of the multigroup diffusion equations that enable one to take into account transport effects. The homogenization methodologies use solution of specially defined transport problems to generate homogenized cross sections for the multigroup diffusion equations. Most of current methodologies (see, for example, Sanchez (2009)) use k-eigenvalue transport problems to calculate averaging shape functions. Recently Dugan et al. (2016) developed advanced homogenization methods apply α-eigenvalue transport problems.

The standard methods of approximate solutions of non-stationary problems are used for modelling of the dynamics of neutron-physical processes. The most attention is paid to two-level schemes with weights (θ-method) (Ascher, 2008; LeVeque, 2007; Hundsdorfer and Verwer, 2003), the Runge–Kutta and Rosenbrock schemes (Butcher, 2008; Hairer and Wanner, 2010) are used. Let's note a special class of methods for modelling of non-stationary neutron transport in diffusion multigroup approximation, which is connected with multiplicative representation of solution — space–time factorization methods and the quasistatic method (Chou et al., 1990; Dahmani et al., 2001; Dodds, 1976; Goluoglu and Dodds, 2001). The approximate solution is searched in the form of the product of two functions, one of which depends on time and is related to the amplitude, the second one (the shape function) describes the spatial distribution. It is difficult to check the accuracy of the approximate solution in such approach, in particular, while calculating the dynamic modes with complicated changes in neutron flux distribution.

The processes occurring in a nuclear reactor are essentially non-stationary. The stationary state of neutron flux, which is related to the critical state of the reactor, is characterised by local balancing of neutron absorption and generation. This boundary state is usually described by solution of a spectral problem (Lambda Modes

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problem, λ-eigenvalue problem) provided that the fundamental eigenvalue (maximal eigenvalue) that is called k-effective of the reactor core, is equal to unity. In this case, the stationary neutron field is related with the corresponding eigenfunction. Calculations of k-effective of the reactor on the basis of the spectral Lambda Modes problem solution are obligatory for developing a new design of reactor installation.

Time behaviour of nuclear reactor is deemed sometimes to be related to the deviation of k-effective from unity that involves, in particular, concept of reactivity. This is not justified, since, while calculating this parameter, the evolutionary nature of neutron redistribution processes (nonstationary systems of the equations) is considered in no way. The k-effective parameter deviates from unity, though quite weakly, but anyway such a solution, generally speaking, cannot be connected with the stationary solution of the problem. There is simply no such a solution. Thus, the attempts to correct the basic mathematical model of non-stationary neutron diffusion by introducing some correcting multipliers to achieve the strict criticality are not successful.

The spectral parameter α, which is not directly connected with k-effective, is proposed to be used instead of k-effective for more adequate characteristic of the dynamic nature of reactor. It is defined as the fundamental eigenvalue of the spectral problem (time-eigenvalue, α-eigenvalue problem), which is connected with the non-stationary equations of neutron diffusion (Bell and Glasstone, 1970; Modak and Gupta, 2007; Verdu et al., 2010). By analogy with the usual problems of heat conductivity (see, for example, Luikov, 1968; Samarskii and Vabishchevich, 1996) we consider the regular reactor mode. At large times the behavior of a neutron flux is asymptotic, and one can talk about space–time factorization solution, whose amplitude is $\exp(\alpha t)$, the shape function is the eigenfunction of the spectral problem.

The Lambda and Alpha Modes spectral problems deal with a not self-adjoint vector elliptic operator. Generally, the eigenvalues are complex. The strict conclusion concerning the eigenvalue reality was obtained (see, for instance, Habetler and Martino, 1961) under reasonable physics assumptions only for the fundamental eigenvalue. Performed precise calculations of the reactor test problems (the VVER-1000 reactor without a reflector and HWR reactor) confirm the fact that the next eigenvalues may be complex with small imaginary parts. Our investigation clarifies the results of other authors (González-Pintor et al., 2009), which give only real parts of the eigenvalue for the same test problems. These clarifications deal with the accuracy control during eigenvalue and eigenfunction calculations using a set of fine meshes and finite elements of different degree; also we used applied software aimed to solve spectral problems with not self-adjoint operators.

Study of the dynamic processes can be based on the discrimination of symmetric and skew-symmetrical parts of the neutron transport operator. In this case, we can easily get the a priori assessments of stability in the corresponding norm, while assessing the operator of the symmetric part from below, and perform the analysis of used time approximations (Samarskii, 2001; Samarskii et al., 2002). To get this, the partial spectral problem is solved to find the fundamental eigenvalue δ of the Delta Modes spectral problem.

The paper is organised as follows. The statement of the boundary-value problem for the system of non-stationary diffusion equations in multigroup approach is given in Section 2. Various spectral problems are discussed in Section 3. A numerical example of calculation of spectral characteristics within the frameworks of two-dimensional test problems for VVER-1000 reactor and HWR reactor using the two-group system of diffusion equations is discussed in Section 4. The results of the work are summarised in Section 5.

2. Problem statement

The neutron flux is considered in multigroup diffusion approximation. The neutron dynamics is considered in the limited convex two-dimensional or three-dimensional area $\Omega(\boldsymbol{x} = \{x_1, \dots, x_d\} \in \Omega,\ d = 2,3)$ with boundary $\partial\Omega$. The neutron transport is described by the system of equations:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} - \nabla \cdot D_g \nabla \phi_g + \Sigma_{rg}\phi_g - \sum_{g \neq g'=1}^{G} \Sigma_{s,g' \to g}\phi_{g'} = (1-\beta)\chi_g \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'} + \widetilde{\chi}_g \sum_{m=1}^{M} \lambda_m c_m, \quad g = 1,2,\dots,G. \tag{1}$$

Here $\phi_g(\boldsymbol{x}, t)$ — neutron flux of g group at point $\boldsymbol{x}$ and time t, G — number of energy groups, v_g — effective velocity of neutrons in the group g, $D_g(\boldsymbol{x})$ — diffusion coefficient, $\Sigma_{rg}(\boldsymbol{x}, t)$ — removal cross-section, $\Sigma_{s,g' \to g}(\boldsymbol{x}, t)$ — scattering cross-section from group g' to group g, β — effective fraction of delayed neutrons, $\chi_g, \widetilde{\chi}_g$ — spectra of prompt and delayed neutrons, $\nu\Sigma_{fg}(\boldsymbol{x}, t)$ — generation cross-section of group g, c_m — density of sources of delayed neutrons of m-type, λ_m — decay constant of sources of delayed neutrons, M — number of types of delayed neutrons. The density of sources of delayed neutrons is described by the equations:

$$\frac{\partial c_m}{\partial t} + \lambda_m c_m = \beta_m \sum_{g=1}^{G} \nu\Sigma_{fg}\phi_g, \quad m = 1,2,\dots,M, \tag{2}$$

where β_m is a fraction of delayed neutrons of m-type, and

$$\beta = \sum_{m=1}^{M} \beta_m.$$

System of Eqs. (1) and (2) is supplemented with corresponding initial and boundary conditions.

The albedo-type conditions are set at the boundary $\partial\Omega$ of the area Ω:

$$D_g \frac{\partial \phi_g}{\partial n} + \gamma_g \phi_g = 0, \qquad g = 1,2,\dots,G, \tag{3}$$

where n — outer normal to the boundary $\partial\Omega$.

Let's propose that the reactor was critical up to the initial time moment ($t = 0$):

$$\phi_g(x, 0) = \phi_g^0(x), \quad c_m(x, 0) = c_m^0(x). \tag{4}$$

For $\phi_g^0(x)$ and $c_m^0(x)$ we get:

$$-\nabla \cdot D_g \nabla \phi_g^0 + \Sigma_{rg}\phi_g^0 - \sum_{g \neq g'=1}^{G} \Sigma_{s,g' \to g}\phi_{g'}^0 = \left((1-\beta)\chi_g + \beta\widetilde{\chi}_g\right) \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'}^0, \quad g = 1,2,\dots,G,$$

$$\lambda_m c_m^0 = \beta_m \sum_{g=1}^{G} \nu\Sigma_{fg}\phi_g^0, \quad m = 1,2,\dots,M.$$

Let's consider the problem without taking into account delayed neutrons (all neutrons are prompt). We assume that all neutrons (including delayed neutrons) are born as prompt, but their spectra χ_g and $\widetilde{\chi}_g$ are different. Then instead of (1) one can obtain the following equation:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} - \nabla \cdot D_g \nabla \phi_g + \Sigma_{rg}\phi_g - \sum_{g \neq g'=1}^{G} \Sigma_{s,g' \to g}\phi_{g'} = \left((1-\beta)\chi_g + \beta\widetilde{\chi}_g\right) \sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'}, \quad g = 1,2,\dots,G. \tag{5}$$