Reliability Engineering and System Safety 132 (2014) 90–96

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Reliability analysis of repairable multi-state system with common bus performance sharing

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ARTICLE INFO

Article history:
Received 17 February 2014
Received in revised form
12 June 2014
Accepted 17 July 2014
Available online 27 July 2014

Keywords:
Instantaneous availability
Multi-state system
Performance sharing
Universal generating function
Stochastic process method
Repairable

ABSTRACT

In this paper, an instantaneous availability model for repairable multi-state system (MSS) with common bus performance sharing is proposed. The repairable MSS consists of some multi-state units and a common bus performance redistribution system. Each unit in the system has several performance levels and must satisfy its individual random demand. A unit can transmit the surplus performance to other units in real time through the common bus performance redistribution system, if it has a performance that exceeds its demand. The entire system fails if the demand of any unit is not satisfied. A new method based on the combination of the stochastic process method and the universal generating function technique is suggested to evaluate the instantaneous availability and the mean instantaneous performance deficiency of the proposed repairable MSS. Two examples are given for applications in the end.

1. Introduction

Performance sharing problem is widely existed in many real-life situations, such as power supply systems, intelligent transportation systems, network and parallel computing systems [1–3]. In general, there are some constraints of resources when a system operates. In order to improve the system, units within a system can share some resources such as performance with each other. In such system, the surplus performance of some units with abundant performance can be shared with other units that experience performance deficiency [4,5], and the system is called a system with performance sharing.

A system that can have a finite number of performance states is called a Multi-State System according to Ref. [6]. Recent researches on MSS are often focused on the reliability evaluation and optimization of MSS [7–16]. A Multi-State System with performance sharing was first studied by Lisnianski and Ding [1], where two multi-states units were taken into account under dynamic state, and the surplus performance could only be transmitted one-way from the reserve unit to the main unit in real time. Then, it was extended to the reliability evaluation of multi-directional performance transmission by Levitin [17], where a common bus model was employed and the MSS was a static system, and it was only focused on the steady-state of the MSS.

In many real-life situations, the state of the MSS is time-varied [18–20], and it is often repaired to improve the system reliability by components repair or replacement [21–25]. Therefore, it is very valuable to evaluate the instantaneous characteristics of the MSS with performance sharing. The repairable components are often involved in the MSS. In this paper, it is assumed to start repair actions immediately after a failure, and the repair time is neglected. In order to evaluate the system availability of MSS, the stochastic process method and the universal generating function (UGF) technique are usually adopted [26–31,40], and their combined method is named as *Lz*-transform by Lisnianski [32]. *Lz*-transform is an effective tool in the reliability analysis of multi-state systems. *Lz*-transform has been applied in availability assessment of an aging multi-state water cooling system for medical equipment by Frenkel et al. [33]. Some other works can be seen in Refs. [34,35].

A repairable Multi-State System (RMSS) with common bus performance sharing consists of some multi-state units and a common bus performance redistribution system, and it is investigated in this paper. Each unit in the RMSS can have different numbers of performance levels, and it must satisfy its own demand. Each unit is connected to the common bus. Through the common bus redistribution system, the units in the situation of performance deficiency can share the surplus performance of the units with abundant performance in real time. Common bus structure is widely used in the computer, network and many other

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http://dx.doi.org/10.1016/j.ress.2014.07.017
0951-8320/

fields. In a bus network, all units (computers) located on the network share the common bus. In order to achieve a common goal, such as solving a large computational problem, they can communicate by passing packages of tasks through the common bus. A problem is usually divided into many tasks. Each unit is assigned to deal with one or more tasks, which can be defined as their own demands. When the processing capacity of a unit exceeds its amount of task, it can be defined as the performance surplus. Whereas, when the processing capacity of a unit does not exceed its amount of task, it can be defined as the performance deficiency. There are many unknowns and uncertainties of a computational problem in the network, such as the arrival time and complexity. Those uncertainties result in the demand of each unit being randomly vary with time. Transmission capacity is largely dependent on the internal material of the common bus. It is also influenced by external environment factors such as temperature and humidity. Therefore, transmission capability also varies with time. In a power generating system, numerous different units are connected to a common bus. The common bus is an electrical conductor that can transmit energy among the units. The units consist of several different generators. The performance of the units can vary depending on power output of the generators. The demand of the units comes out of the energy consumed by the units. It also changes with the time and is uncertain. Each unit must satisfy its individual random demand. All the units can transmit their surplus energy to other units in the situation of performance deficiency. Further more, the application of the common bus in power generating system always results in the cost savings, maintenance reduction and reliability improvement. The entire system fails if the demand of any unit is not satisfied. The system instantaneous availability $A(t)$ at any instant $t>0$ is defined as the probability that the entire system meets all the units demand. The instantaneous availability and the mean instantaneous performance deficiency of the proposed RMSS are investigated and the *Lz*-transform method is used to give the instantaneous characteristics of the proposed MSS.

In this paper, the RMSS model with common bus performance sharing is presented in Section 2. In Section 3, *Lz*-transform is adopted to evaluate the system instantaneous availability and its mean instantaneous performance deficiency. Two illustrative examples are given for application in Section 4. Some conclusions are given in the end.

2. Model descriptions

The RMSS with common bus performance sharing consists of N multi-state units and a common bus performance redistribution system. Its structure is shown in Fig. 1. Each unit of the system is repairable and multi-state. They can share the surplus performance in real time through the common bus performance redistribution system.

The unit i has K_i different performance levels. The set $\mathbf{g}_i=\{g_{i,1},\ldots,g_{i,K_i}\}$ stands for the set of performance levels corresponding to the unit i. In the set, performance levels are in ascending order. The level K_i is associated with the nominal performance level $G_i(t)=g_{i,K_i}$, and the level 1 is associated with complete failure $G_i(t)=g_{i,1}=0$. All other levels $j\in\{2,\ldots,K_i-1\}$ are associated with corresponding reduced capacity levels $g_{i,j}$. At any instant $t>0$, the performance level $G_i(t)$ of the unit i takes some value from the set $\mathbf{g}_i$, namely, $G_i(t)\in\mathbf{g}_i$. As pointed by Lisnianski et al. [1], the performance level $G_i(t)$ can be defined as a discrete-state continuous-time (DSCT) Markov stochastic process, where the process behavior at a future instant only depends on the current state. The Markov model of a multi-state unit was introduced by Lisnianski et al. [36], where minor and major failures/repairs of units were involved. The imperfect maintenance model for multi-state units was recommended by Pandey et al. [37], where component replacement and imperfect repair/maintenance were investigated. Component replacement means that a new component is installed in place of the old one. After replacement, the unit i returns the highest performance level $K_i(G_i(t)=g_{i,K_i})$. When the imperfect repair is done, the performance level of the unit i is improved from complete failure $(G_i(t)=0)$ or middle-level $0=g_{i,1}<G_i(t)<g_{i,K_i}$ to a higher performance level (other than g_{i,K_i}).

The unit i must satisfy its H_i different demand levels. The set $\mathbf{w}_i=\{w_{i,1},\ldots,w_{i,H_i}\}$ stands for the set of demand levels corresponding to the unit i. In the set, demand levels are in ascending order. The demand level of the unit i can also be presented as a DSCT Markov stochastic process $W_i(t)$. At any instant $t>0$, the demand level $W_i(t)$ of the unit i takes some value from the set $\mathbf{w}_i$, namely, $W_i(t)\in\mathbf{w}_i$.

Transmission capacity of the common bus performance redistribution system is changing over time. It may operate in L levels. The set $\mathbf{c}=\{c_1,\ldots,c_L\}$ stands for the set of the transmission levels corresponding to the unit i. In the set, transmission levels are in ascending order. Similarly, the transmission capacity can also be described as a DSCT Markov stochastic process $C(t)$ as well. At any instant $t>0$, the transmission capacity $C(t)$ takes some value from the set $\mathbf{c}$, namely, $C(t)\in\mathbf{c}$.

In order to study the system behavior, units behavior of the system should be investigated at first. The unit performance level, the unit demand level and the transmission capacity can be described by the DSCT Markov stochastic process. For example, the unit performance level is analyzed. The multi-state Markov model is shown in Fig. 2. Failures cause the unit performance transition from a upper state l to a lower state j $(j<l)$ with the corresponding transition intensity $\lambda^i_{l,j}$. Repairs return the unit performance transition from lower state j to upper state l $(j<l<K_i)$ with the corresponding transition intensity $\mu^i_{j,l}$.

A unit will send the surplus performance to other units through the performance redistribution system, if its performance exceeds the needed demand. That is to say, one unit can share the surplus performance from other units, if its performance does not meet its

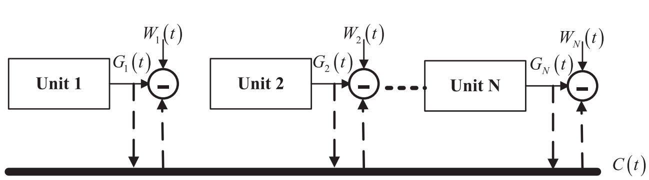

Fig. 1. The structure of the RMSS system with common bus performance sharing.

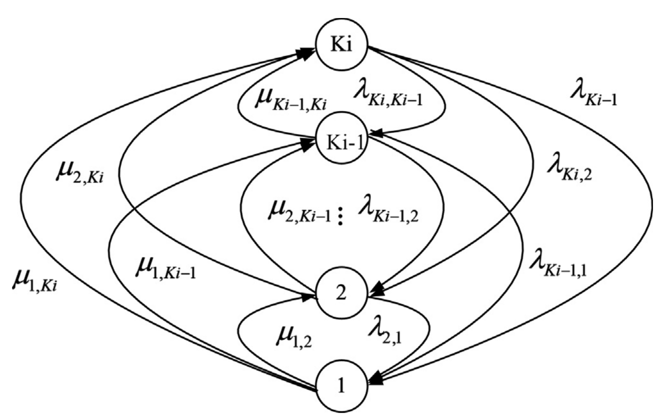

Fig. 2. The multi-state Markov model for the unit i.