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Uncertainty propagation and sensitivity analysis in system reliability assessment via unscented transformation

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ABSTRACT

The reliability of a system, notwithstanding it intended function, can be significantly affected by the uncertainty in the reliability estimate of the components that define the system. This paper implements the Unscented Transformation to quantify the effects of the uncertainty of component reliability through two approaches. The first approach is based on the concept of uncertainty propagation, which is the assessment of the effect that the variability of the component reliabilities produces on the variance of the system reliability. This assessment based on UT has been previously considered in the literature but only for system represented through series/parallel configuration. In this paper the assessment is extended to systems whose reliability cannot be represented through analytical expressions and require, for example, Monte Carlo Simulation.

The second approach consists on the evaluation of the importance of components, i.e., the evaluation of the components that most contribute to the variance of the system reliability. An extension of the UT is proposed to evaluate the so called "main effects" of each component, as well to assess high order component interaction. Several examples with excellent results illustrate the proposed approach.

1. Introduction

Reliability is defined as in [1]: the probability that a system, subsystem or device will perform adequately for a specified period of time under specific operating conditions.

Commonly, for system users, the reliability of a system contributes significantly to the perception of its performance or effectiveness. For years, system reliability analysis has been an essential design component in a myriad of systems, from small electronics to large-scale systems with multiple interactions; among others the power electrical system, the cellular telephone system, etc. [2]. In recent decades competition in the global market and quality requirements from users and industry standards, have made reliability analysis an important tool in virtually all fields production.

The reliability of systems depends on system configuration, service function and the reliability of system components. Sometimes the system configuration and its service function (for example, the connectivity between two points) allow obtaining an analytic expression that facilitates calculating system reliability. This is the case of systems that can be modeled with components configured in series, parallel or mixture of these configurations. In other cases, the configuration does not allow modeling the reliability via a closed form expression. For example, components that are neither in series nor in parallel are defined in the literature as complex systems reliability [1,2] and could require computationally intensive procedures (e.g. determining the minimum cut sets [1]). Further, in some cases, the system performance/service function poses additional modeling problems. This is the case, for example, of systems that requires specific operational constraints, such as, the flow between two points in the system. In this case, Monte Carlo simulation could be used [1].

Whatever the case, the system reliability is a function of the system state vector $\mathbf{v}=(v_1, v_2, \ldots, v_n)$ denoting the state of a system with n components (either working or not). The function $\varphi: \Re^n \rightarrow \Re^+$ maps the system state vector into a system state. That is, $\varphi(\mathbf{v})$ is the performance service function under system state vector $\mathbf{v}$. Then R_s denotes the probability that the system is operating properly, for binary systems: $R_s=P(\varphi(\mathbf{v})=1)$. Furthermore it is assumed that $R_i=P(v_i=1)$. Thus system reliability is a function of the reliability of its components and can be expressed as $R_s=R(R_1, R_2,\ldots, R_n)$, where $R_s()$ (the output) is a function that considers the configuration and the type of system performance (e.g., continuity or maximum flow between two nodes). The function $R()$ could be evaluated through an analytical expression (e.g., elements in series/parallel configuration) or a black-box model (e.g., a Monte Carlo simulation).

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System reliability is affected largely by the uncertainty that may exist in the reliability system components. If the component reliability is a random variable with known probability function then $R_s()$, is a random variable with a probability function, generally unknown. The effect of the uncertainty of components in the reliability of the system, usually evaluated through its variance [3,4] could suggest to a decision-maker to try to limit these variations, for example, during system design. According to [5], there are two basic approaches to estimate the effect that causes the uncertainty of the input variables in the output, as well as knowing which inputs have the greatest impact on the system output variation:

- Sensitivity analysis allows determining the effect of each input variable, or group of variables, in the system output; and
- Uncertainty propagation allows assessing the changes generated in the output due to changes in the inputs.

Several methods for propagation of uncertainty have been suggested, like: interval arithmetic [6,7], Taylor series [5,8], Moments [5,8,9], Monte Carlo simulation [5,8,9], among others. Perhaps Monte Carlo simulation is the most used method. The method requires information on the type of probability distribution associated to inputs, a mechanism for generating input deviates, and a function that allows the evaluation of such deviates. As a result, an approximate distribution function is derived, which allows estimating several properties of the output, such as mean and variance.

In the context of this paper, the Monte Carlo method yields an estimate of the distribution of the reliability function $R_s()$. However, to obtain acceptable results, $R_s(\cdot)$ must be evaluated on a set of m samples $(R_1, R_2, \ldots R_n)_j$, $j=1,\ldots m$, with $m \gg n$ [10].

Several authors have analyzed the uncertainty propagation in reliability systems. For example Coit [3] derived closed expressions for systems in series-parallel configuration or systems that can be decomposed into a series-parallel configuration via minimal cut sets. Jin and Coit [11] studied the system reliability variance "when components can be repeatedly used in the system and reliability estimates may not be independent". In [4] the authors presented new strategies to improve the estimation of confidence interval estimates for system reliability, based on the reliability of the components and their associated uncertainty, measured through variance [3]. The study is limited to series – parallel systems, with two-state components, without capacity constraints and considering statistical independence. In [12] the authors present an approach for determining lower and upper bounds for the reliability between two nodes of a system to meet a demand d, known as the 2TRD problem. The elements are modeled as two-state with capacity constraints. This approach assumes the independence of the elements and requires knowledge of the set of minimal paths (a problem that can be computationally complex [13]).

To cope with these limitations, a new technique for uncertainty propagation was proposed in [14]. The approach, called the Unscented Transform (UT), is based on the idea that it is "easier to approximate a probability distribution than to approximate an arbitrary nonlinear function or transformation" [14].

As explained in detail in [14], the UT provides a more direct mechanism for obtaining the mean and covariance of a set of variables $\mathbf{X}=(x_1, x_2,\ldots, x_n)$ that undergoes a non-linear transformation $\mathbf{f}(\mathbf{X})$. This mechanism is based on: (a) the definition of a set of points in the input space, called *sigma points*; (b) their non-linear transformation; and (c) a simple procedure to approximate the mean and covariance of the transformed set of points.

The definition of the sigma points is done deterministically, by considering certain conditions. For example, the mean and covariance of the sigma points must match the mean and covariance of the variables to be transformed. Special attention should be placed to the fact that the sigma points must belong to the feasible region of the problem under study. In our case, the variables $(x_1, x_2,\ldots, x_n)$ represent the reliability of components and therefore each must belong to [0,1].

There are several methods for selecting the sigma points: standard UT, scaled UT, among others [14–18]. Some of the methods are compared in [19]. In any case, the number of sigma points to be considered is linearly proportional to the number of system variables, but much less than the number of evaluations required by the Monte Carlo technique. The UT has been successfully applied in different fields, like control [14], power systems [20], robust design [21], micro electromechanical systems [22], electromagnetic simulations [23,24], medical statistic [25], Prognostics of Lumen Maintenance [26], option pricing [27], among others.

In [28] the author applies the standard UT to estimate reliability of a series-parallel binary system without capacity constraints. While the author claims that the use of classical UT is very promising, he did not consider some numerical details. Most importantly, the fact that the method selected for defining the sigma points generate values outside the range [0, 1].

In summary, the previous studies have in common that the elements of the system are independent, are only modeled as two-state, and most of them consider as a performance function the continuity of the flow between two points of the system. This paper aims to extend the use of the Unscented Transform for estimating the expected value and variance associated to the reliability of various systems, with reliability known function expressed analytically (e.g., series – parallel configurations) or estimated from a standard Monte Carlo simulation, binary or multistate elements, with capacity constraints or under any another performance function and without the assumption of independent elements. Additionally, an extension is suggested to determine the importance of the elements, or groups of elements, on system reliability variations due to uncertainty propagation. To our knowledge, this assessment has not been previously analyzed.

The rest of the paper is organized as follows: Section 2 describes the Unscented Transformation. Section 3 suggests a general procedure for applying UT in reliability assessment while Section 4 proposes the derivation of importance indexes. Section 5 describes several examples. Finally, Section 6 shows the conclusions and work in progress.

2. The unscented transformation

2.1. Basic technique

The UT rationale is that it is "easier to approximate a probability distribution than to approximate an arbitrary nonlinear function or transformation" [14].The approach is based on selecting a set of points (sigma points) so that their mean and covariance match the mean and covariance of a selected distribution (not necessarily Gaussian type). Each sigma point is then transformed in a cloud of points that allows the estimation of the mean and covariance of the transformation.

Let $\mathbf{X}$ be a vector of n-dimensional input random variables with mean $\overline{\mathbf{X}}$ and $\mathbf{P_{XX}}$ the variance-covariance matrix, that allows relating the input variable dependence.

Let $\mathbf{Y}$ be a vector of r-dimensional non-linear functions that maps the input space, i.e., $\mathbf{Y}=\mathbf{f}(\mathbf{X})$, $\mathbf{f}$: $\mathbb{R}^n \rightarrow \mathbb{R}^r$. For reliability evaluations, the n-dimensional input random variables correspond to the component reliabilities while $\mathbf{Y}$ is defined as a single function R_s. For example, for two components in series configuration, $Y=R_1 * R_2$. It is important to mention that in many situations the function to evaluate the system reliability is not expressed analytically and approximations are required, for example using a MCS. As mentioned, this situation does not pose a problem under the UT approach.