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Effect of the time variation of the neutron current density in the calculation of the reactivity

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ABSTRACT

In the derivation of the conventional set of point kinetics equations from the neutron transport equation, one of the approximations used is to disregard the time variation of the neutron density current in comparison with the other terms resulting from the P1 approximation. When not considering this approximation it is obtained a modified set of point kinetics equations where new terms appear naturally during the derivation. In this paper, the effect from considering the time variation of a neutron density current in the calculation of reactivity is evaluated. The new expression obtained can be written as a correction function $\Delta\rho(t)$ to the reactivity conventionally calculated. For the different types of transients simulated with the use of typical kinetic parameters in PWR reactors it was seen that, for those that have an exponential behaviour, the correction $\Delta\rho(t)$ quickly reaches values that cannot be neglected. Additionally, for this type of transient, it is possible to conclude that the reactivity calculated by the conventional expression overestimates that which is obtained by the new formulation by a value proportional to the argument of the exponential function.

1. Introduction

The nuclear power distribution in a nuclear reactor implies investigating the neutron transport in a heterogeneous media and with a strong neutron absorption, and these neutrons can also be scattered by the target cores or escape from the active part of the reactor. The approximation for neutron diffusion is still and largely used in stationary calculations to predict the neutron distribution and the critical boron concentration, despite the advances in computing that allow solving the neutron transport equation through several methods. To deal with the movement of neutrons in a manner analogous to that of heat diffusion requires the execution of several approximations in the transport equation which includes a weak angular dependence on the angular distribution of the neutrons, isotropic neutron sources, and disregarding the derivative for the neutron current density, in comparison with other terms that appear in the neutron transport equation (Duderstadt and Hamilton, 1976).

The transient situations found in a nuclear reactor can be predicted only through the modification of the neutron flux and, as a result, it is possible to make a sufficiently precise forecast on the consequences of the transients. It is enough to relate the magnitude of the neutron flux, which varies in time, with the neutron population in the core of a nuclear reactor (Henry, 1975). Point kinetics equations relate these parameters and thus allow a study of the transient situations that may occur in a nuclear reactor, and their obtaining takes place from a sequence of approximations, done from the neutron transport theory. Their obtaining can be accomplished directly from the neutron transport equation, for the neutron diffusion equation, or through a heuristic procedure, according to Stacey (2007) and Henry (1975).

Some changes in the point kinetics equations have recently been proposed. In Espinosa-Paredes et al. (2008) and Espinosa-Paredes et al. (2011) a derivation is made of a model for fractional point kinetics that uses equations for point kinetics with derivative terms of a non-integer order, that is, adopting fractional derivatives (Aboanber and Nahla, 2016; Espinosa-Paredes, 2016; Altahhan et al., 2016). The calculation for the case of inserting a sinusoidal reactivity for this model of fractional point kinetics is done in the paper of Polo-Labarrios et al. (2014). In Quintero-Leyva (2015, 2016) a numerical solution is obtained for an integral–differential formulation for classical point kinetics, eliminating the concentration of precursors and thus obtaining one single equation with one single variable, even when considering six groups of precursors. However, this paper will adopt the point kinetics formulation proposed by Nunes et al. (2015) where the neutron current derivative is not disregarded.

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Several papers are considered in the attempt to obtain the analytical solutions for classical point kinetics equations (Gonçalves et al., 2015; Schramm et al., 2016; Palma et al., 2009) for the case of a linear variation in reactivity during the start-up procedure of a nuclear reactor. An analytical representation is determined by Leite et al. (2014) for classical point kinetics equations with an adapted time step. In Razak et al. (2015) the solution for point kinetics equations is obtained via the method of differentiation for temporal exponentials from a Taylor series expansion.

In practice the use of point kinetics equations takes place in the so-called inverse kinetics where the reactivity is obtained from a nuclear power history (Duderstadt and Hamilton, 1976). There are only a few problems for which it is possible to obtain an exact analytical solution for neutron density given a specific reactivity. Indeed, it is frequently more appropriate to invert the problem calculating the reactivity that will determine the past behaviour for the neutron density expressed from a direct relation with the nuclear power. This procedure is more aligned with the nuclear reactor control methodology according to Henry (1975) and Suzuki and Tsunoda (1964).

Inverse kinetics provides a promising scenario considering some relevant papers (Suescun et. al., 2012; Suescun et. al., 2013; Antolin et. al., 2013; Malmir and Vosoughi, 2013). In Díaz et al. (2008) a calculation for reactivity was performed using the Laplace transformation and the FIR Filter. The inverse point kinetics equation is therefore solved from a method that uses a discrete version of the Laplace transform. A new method to obtain reactivity is proposed in the paper of Shimazu (2014), its main features being robustness and simplicity, without the use of complex filters.

The goal of this paper consists on obtaining the reactivity for a power history, considering the formulation proposed by Nunes et al. (2015).

2. Modified point kinetics equations

The neutron transport theory is largely used to describe the neutron flux in a nuclear reactor, described in terms of the angular flux $\varphi(\vec{r}, E\hat{\Omega}, t)$ as follows, and according to Duderstadt and Hamilton (1976), Stacey (2007) and Henry (1975).

$$\frac{1}{\mathrm{v}(E)}\frac{\partial\varphi(\vec{r},E,\hat{\Omega},t)}{\partial t}+\hat{\Omega}\cdot\vec{\nabla}\varphi(\vec{r},E,\hat{\Omega},t)+\sum_t(\vec{r},E,t)$$
$$=\frac{1}{4\pi}(1-\beta)\chi_P(E)\int_{4\pi}\int_0^\infty \upsilon(E')\sum_f(\vec{r},E',t)\varphi(\vec{r},E',\hat{\Omega},t)dE'd\hat{\Omega}'$$
$$+\int_{4\pi}\int_0^\infty\sum_S(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega},t)\varphi(\vec{r},E',\hat{\Omega},t)dE'd\hat{\Omega}'$$
$$+\frac{1}{4\pi}\sum_{i=1}^{6}\lambda_i\chi_i(E)C_i(\vec{r},t) \tag{1}$$

$$\frac{\partial C_i(\vec{r},t)}{\partial t}=\beta_i\int_{4\pi}\int_0^\infty \upsilon(E')\sum_f(\vec{r},E',t)\varphi(\vec{r},E',\hat{\Omega},t)dE'd\hat{\Omega}'$$
$$-\lambda_iC_i(\vec{r},t), \tag{2}$$

where the parameters are defined as follows: $C_i(\vec{r},t)$ is the concentration of precursors, $v(E)$ is the speed of the neutrons, $\hat{\Omega}$ is the unit vector towards the displacement of the neutrons, $d\hat{\Omega}$ is the unit solid angle, $\sum_t(\vec{r},E,t)$ is total neutron cross section, $\sum_S(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega},t)$ is the scattering kernel, $\sum_f(\vec{r},E',t)$ is the fission cross section, $\upsilon(E')$ is the average number of neutrons produced by fission caused by a neutron with energy E, $\chi_i(E)dE$ is the mean number of neutrons produced in the fission that are born with energy E in dE, β is the delayed neutrons fraction in relation to total neutrons, β_i is the share of delayed neutrons in the group i of precursors in relation to total neutrons, and λ_i is the decay constant for precursor neutrons in group i.

Using the P1 approximation, which consists of expanding the scattering kernel in Legendre polynomials to the second degree as follows,

$$\Sigma_S(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega},t)\cong\sum_{l=0}^{1}\frac{2l+1}{4\pi}\Sigma_{Sl}(\vec{r},E'\to E,t)P_l(\hat{\Omega}'\cdot\hat{\Omega})$$
$$=\frac{1}{4\pi}\Sigma_{S0}(\vec{r},E'\to E,t)+\frac{3}{4\pi}\Sigma_{S1}(\vec{r},E'\to E,t), \tag{3}$$

and replacing in Eq. (1), we obtaining:

$$\frac{1}{\mathrm{v}(E)}\frac{\partial\varphi(\vec{r},E,\hat{\Omega},t)}{\partial t}+\hat{\Omega}\cdot\vec{\nabla}\varphi(\vec{r},E,\hat{\Omega},t)+\sum_t(\vec{r},E,t)$$
$$=\frac{1}{4\pi}(1-\beta)\chi_P(E)\int_{4\pi}\int_0^\infty \upsilon(E')\sum_f(\vec{r},E',t)\varphi(\vec{r},E',\hat{\Omega},t)dE'd\hat{\Omega}'$$
$$+\frac{1}{4\pi}\int_{4\pi}\int_0^\infty\Sigma_{S0}(\vec{r},E'\to E,t)\varphi(\vec{r},E',\hat{\Omega},t)dE'd\hat{\Omega}'$$
$$+\frac{3}{4\pi}\int_{4\pi}\int_0^\infty\Sigma_{S1}(\vec{r},E'\to E,t)\varphi(\vec{r},E',\hat{\Omega},t)dE'd\hat{\Omega}'$$
$$+\frac{1}{4\pi}\sum_{i=1}^{6}\lambda_i\chi_i(E)C_i(\vec{r},t). \tag{4}$$

After that one applies the operator $\int_{4\pi}(\cdot)d\hat{\Omega}$ to Eqs. (4) and (2), to obtain the spatial kinetics equations, that are:

$$\frac{1}{\mathrm{v}(E)}\frac{\partial}{\partial t}\int_{4\pi}\varphi(\vec{r},E,\hat{\Omega},t)d\hat{\Omega}+\int_{4\pi}\hat{\Omega}\cdot\vec{\nabla}\varphi(\vec{r},E,\hat{\Omega},t)d\hat{\Omega}$$
$$+\Sigma_t(\vec{r},E,t)\int_{4\pi}\varphi(\vec{r},E,\hat{\Omega},t)d\hat{\Omega}$$
$$=\frac{1}{4\pi}\sum_{i=1}^{N}\int_{4\pi}\lambda_i\chi_i(E)C_i(\vec{r},t)d\hat{\Omega}$$
$$+\frac{1}{4\pi}(1-\beta)\chi_P(E)\int_{4\pi}\int_{4\pi}\int_0^\infty\upsilon(E')\Sigma_f(\vec{r},E',t)\varphi(\vec{r},E',\hat{\Omega}',t)dE'd\hat{\Omega}'d\hat{\Omega}$$
$$+\frac{1}{4\pi}\int_{4\pi}\int_{4\pi}\int_0^\infty\Sigma_{S0}(\vec{r},E'\to E,t)\varphi(\vec{r},E',\hat{\Omega}',t)dE'd\hat{\Omega}'d\hat{\Omega}$$
$$+\frac{3}{4\pi}\int_{4\pi}\int_{4\pi}\int_0^\infty\Sigma_{S1}(\vec{r},E'\to E,t)\varphi(\vec{r},E',\hat{\Omega}',t)\hat{\Omega}'\cdot\hat{\Omega}dE'd\hat{\Omega}'d\hat{\Omega}, \tag{5}$$

and

$$\frac{\partial}{\partial t}\int_{4\pi}C_i(\vec{r},t)d\hat{\Omega}=\beta_i\int_{4\pi}\int_{4\pi}\int_0^\infty\upsilon(E')\Sigma_f(\vec{r},E',t)\varphi(\vec{r},E',\hat{\Omega}',t)dE'd\hat{\Omega}'d\hat{\Omega}$$
$$-\int_{4\pi}\lambda_iC_i(\vec{r},t)d\hat{\Omega}. \tag{6}$$

The following are defined:

$$\phi(\vec{r},E,t)\equiv\int_{4\pi}\varphi(\vec{r},E,\hat{\Omega},t)d\hat{\Omega}=\text{neutron flux}$$

$$\vec{J}(\vec{r},E,t)\equiv\int_{4\pi}\varphi(\vec{r},E,\hat{\Omega},t)\hat{\Omega}d\hat{\Omega}=\text{neutron current density}$$

Eqs. (5) and (6) are re-written from the definitions of neutron flux and neutron current density, as follows:

$$\frac{1}{\mathrm{v}(E)}\frac{\partial\phi(\vec{r},E,t)}{\partial t}+\Sigma_t(\vec{r},E,t)\phi(\vec{r},E,t)$$
$$=\int_0^\infty\Sigma_{S0}(\vec{r},E'\to E,t)\phi(\vec{r},E',t)dE'$$
$$+(1-\beta)\chi_P(E)\int_0^\infty\upsilon(E')\Sigma_f(\vec{r},E',t)\phi(\vec{r},E',t)dE'-\vec{\nabla}\cdot\vec{J}(\vec{r},E,t)$$
$$+\sum_{i=1}^{N}\lambda_i\chi_i(E)C_i(\vec{r},t), \tag{7}$$