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Relevance of control theory to design and maintenance problems in time-variant reliability: The case of stochastic viability

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ABSTRACT

The goal of this paper is twofold: (1) to show that time-variant reliability and a branch of control theory called stochastic viability address similar problems with different points of view, and (2) to demonstrate the relevance of concepts and methods from stochastic viability in reliability problems. On the one hand, reliability aims at evaluating the probability of failure of a system subjected to uncertainty and stochasticity. On the other hand, viability aims at maintaining a controlled dynamical system within a survival set. When the dynamical system is stochastic, this work shows that a viability problem belongs to a specific class of design and maintenance problems in time-variant reliability. Dynamic programming, which is used for solving Markovian stochastic viability problems, then yields the set of design states for which there exists a maintenance strategy which guarantees reliability with a confidence level β for a given period of time T. Besides, it leads to a straightforward computation of the date of the first outcrossing, informing on when the system is most likely to fail. We illustrate this approach with a simple example of population dynamics, including a case where load increases with time.

1. Introduction

This paper connects two lines of research, viability and reliability, that have ignored each other up to now despite strong similarities. Both frameworks study the potential for a system to retain desirable properties. They were developed in different contexts and sometimes tackle different specific technical or conceptual issues in relation with the same type of problems, which makes their confrontation promising. In particular, this work focuses on showing how concepts and methods coming from the so-called stochastic viability framework [1] are applicable to time-variant reliability. Indeed, they foster the resolution of a particular class of design and maintenance problems, that this paper is to describe with accuracy.

Reliability theory initially comes from the field of mechanical and structural engineering [2] and has a wide range of applications, from material science [3] and industrial maintenance [4] to ecology [5], environmental management [6] and hydrology [7]. In these applications, different numerical methods enable the estimation of the response surface and the associated probability of a system to be in the so-called failure set. Reliability methods provide ever-improving approximations of this probability of failure in cases of growing complexity, and have been perfected and tailored to an increasing number of applications [8,2,9]. Let us cite for instance Monte Carlo methods, First and Second Order Reliability Methods (FORM and SORM), or response surface approximations. A central concern is often with understanding and modeling the correlations between the different variables.

However, many of these developments deal with time-invariant systems, since they are carried out under a single definite period of time. When the system under consideration evolves in time, the reliability problem is referred to as time-variant. The central issue of representing the correlations between variables is then extended to account for the time correlations of the processes of interest. The probability of reaching the failure set during the evolution is called the cumulative probability of failure. Rice's formula [10], which counts the average number of times an ergodic stationary process crosses a given fixed level, serves as a basis for computing the cumulative probability of failure in the outcrossing approach. This approach is based on the computation and time integration of the outcrossing rate, i.e., the rate at which the state reaches the failure set, e.g. [11]. It has been applied to simple cases where analytical derivations are tractable [12,13] or alongside approaches from time-invariant reliability such as FORM [14], or finite elements methods [15].

Thus, bridges exist between the time-variant and -invariant cases. In fact, some outcrossing algorithms decompose the time-variant problem into a series of time-invariant ones [16,17,13], and

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conversely, the outcrossing rate has been defined on variables other than time [15]. Some cases can even be solved both with the outcrossing rate approach, and by having time as a parameter [18]. Other studies treat a time-variant problem like a time-invariant one, by considering time as a parameter [19,20] or as yet another space variable [21], or by treating a finite number of dates like a series system [22].

Most of the works cited in the two paragraphs above assume a monotonic decrease of performance with time. Such an assumption is perfectly reasonable for structures that deteriorate as they grow old, but recent time-variant reliability studies have questioned its systematic use, and suggest using methods that do not require this hypothesis [23,24,21]. A second limitation of the existing literature, linked with the assumption of a monotonic decrease in performance through time, is the idea that maintenance is the fact of choosing between a limited set of options which essentially are equivalent to rejuvenating the system, e.g. [12,14,25]. Without a monotonic decrease of performance, other types of maintenance need to be taken into account. Besides, there is no framework within the time-variant reliability literature that formally considers design and maintenance together. Nevertheless, design and maintenance are closely related, since a system should be designed in a way that allows for an appropriate maintenance throughout its lifetime.

To address these current limits of time-variant reliability, this work uses a stochastic controlled dynamical system formulation. Non-controlled dynamics can be found in the time-variant reliability literature [26,27,24], and the use of controls leads to a general formulation for design and maintenance problems by linking the acceptability of a design to the existence of a maintenance strategy such that reliability is guaranteed with a confidence β, i.e., such that the cumulative probability of failure is smaller than $1-\beta$.

The link between the initial configuration of a system and the existence of strategies that keep it out of a failure state are central to viability theory [28,29]. This is a control theory that deals with controlled dynamic systems under state constraints, and whose original focus is on controlled deterministic systems. An emphasis is put on finding the viability kernel, the set of all initial states which can be controlled so that their trajectory is maintained within the constraint set at all times. Viability algorithms generally yield both the viability kernel and the associated viable controls at once, e.g. [30–32]. Viability tools have been successfully applied to a variety of fields such as finance, robotics, or ecology, e.g. [33]. Recent work has extended the framework of viability theory in discrete time by considering uncertainties in the dynamics, leading to the definition of the stochastic viability kernel [34], a set of states for which the respect of the constraints can be guaranteed with a desired minimal probability and for a desired time frame. Dynamic programming can compute stochastic viability kernels and determine the control strategy that maximizes the probability to maintain the system in the constraint set during that period [1]. This is the specific development which applicability to reliability problems we propose to demonstrate throughout this work.

The paper is organized as follows. Section 2 introduces the notion of reliability kernel to describe a time-variant design problem. Then Section 3 extends this notion to a coupled problem of design and maintenance through a controlled dynamical system formulation. After that, Section 4 shows how the framework of viability theory applies to a specific case of this coupled design and maintenance problem, and solves it in the Markovian case. Section 5 proposes an application in order to illustrate how dynamic programming can be applied to a reliability problem. The discussion of Section 6 further argues about the potential of confronting reliability with control theories such as viability. Finally, Section 7 summarizes the findings.

2. A design problem in time-variant reliability

This section proposes a general formulation for design problems in time-variant reliability, which comes from a similar problem in time-invariant reliability.

2.1. Time-invariant reliability

Let us consider a system and a vector of n random variables $\mathbf{X}$ which represents the system's state variables and their uncertainty. Reliability is concerned with the performance function $g(\mathbf{X})$, and with the so-called limit-state (or failure) surface defined by [8,9]:

$$g(\mathbf{X})=0 \tag{1}$$

The limit-state surface separates the failure domain F (where $g(\mathbf{X})<0$) from the survival domain S (where $g(\mathbf{X})\geq 0$). The object of reliability is to determine the probability of failure p_f of the system:

$$p_f=\mathbb{P}(\mathbf{X}\in F)=\mathbb{P}(g(\mathbf{X})<0)). \tag{2}$$

A diversity of methods have been developed to compute or approximate the limit-state surface and the probability of failure in the time-invariant case [8,9].

Choices regarding the design of the system may influence the random vector $\mathbf{X}$ or the performance function. Without loss of generality, the problem can be formulated so these choices only affect the former. Let us represent choices by a fixed vector π chosen in a space $\Pi\subset\mathbb{R}^m$ and $m\in\mathbb{N}$. Let us call design this vector: each design leads to a distinct random vector $\mathbf{X}(\pi)$. Then the associated probability of failure $p_f(\pi)$ is

$$p_f(\pi)=\mathbb{P}(g(\mathbf{X}(\pi))<0)). \tag{3}$$

This work focuses on finding values of π such that the system is reliable with a confidence level β (i.e., a significance level $\alpha=1-\beta$). In other words, we are interested in finding elements from the set of design choices such that reliability is achieved with a confidence β. Let us introduce this set as the reliability kernel, noted $\mathrm{Rel}_\pi(\beta)$ and formally written as follows:

$$\mathrm{Rel}_\pi(\beta)=\{\pi\in\Pi | p_f(\pi)\leq 1-\beta\} \tag{4}$$

For instance, $\mathrm{Rel}_\pi(0.99)$ is the set of available designs such that the system has a 99% chance of being in the survival set S. Let us now extend this design problem to the time-variant case.

2.2. Time-variant reliability

We now place ourselves between an initial date $t_0=0$ and final date T, so that the problem is studied within a time interval $[0,T]$ called the planning period. The uncertainty and stochasticity of the system are represented at all dates by the vector $\mathbf{X}(t,\pi)$. There is a consensus in the reliability literature that $\mathbf{X}(t,\pi)$ aggregates a vector of random variables like in time-invariant viability, as well as a vector of one-dimensional random processes that may be correlated with one another as well as with the random variables [14,17,35]. These processes may also be autocorrelated in time.

The performance of the system may also evolve with time, and is now noted $g(t,\mathbf{X}(t,\pi))$. Likewise, the limit-state surface $g(t,\mathbf{X}(t,\pi))=0$ may be dependent on time, and so may the failure domain $F(t)$ (where $g(t,\mathbf{X}(t,\pi))\leq 0$) and the survival domain $S(t)$ (where $g(t,\mathbf{X}(t,\pi))\geq 0$).

Time-variant reliability is concerned with the cumulative probability of failure $p_f(t,\pi)$, the probability of reaching the failure set over $[0,t]$:

$$p_f(t,\pi)=\mathbb{P}(\exists\tau\in[0,t],\mathbf{X}(\tau,\pi)\in F(\tau)) \tag{5}$$