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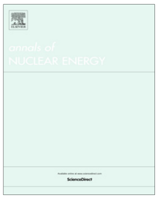

Correlation of errors in the Monte Carlo fission source and the fission matrix fundamental-mode eigenvector

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ABSTRACT

Previous studies raised a question about the level of a possible correlation of errors in the cumulative Monte Carlo fission source and the fundamental-mode eigenvector of the fission matrix. A number of new methods tally the fission matrix during the actual Monte Carlo criticality calculation, and use its fundamental-mode eigenvector for various tasks. The methods assume the fission matrix eigenvector is a better representation of the fission source distribution than the actual Monte Carlo fission source, although the fission matrix and its eigenvectors do contain statistical and other errors. A recent study showed that the eigenvector could be used for an unbiased estimation of errors in the cumulative fission source if the errors in the eigenvector and the cumulative fission source were not correlated. Here we present new numerical study results that answer the question about the level of the possible error correlation. The results may be of importance to all methods that use the fission matrix.

New numerical tests show that the error correlation is present at a level which strongly depends on properties of the spatial mesh used for tallying the fission matrix. The error correlation is relatively strong when the mesh is coarse, while the correlation weakens as the mesh gets finer. We suggest that the coarseness of the mesh is measured in terms of the value of the largest element in the tallied fission matrix as that way accounts for the mesh as well as system properties. In our test simulations, we observe only negligible error correlations when the value of the largest element in the fission matrix is about 0.1. Relatively strong error correlations appear when the value of the largest element in the fission matrix raises above about 0.5.

We also study the effect of the error correlations on accuracy of the eigenvector-based error estimator. The numerical tests show that the eigenvector-based estimator consistently underestimates the errors in the cumulative fission source when a strong correlation is present between the errors in the fission matrix eigenvector and the cumulative fission source (i.e., when the mesh is too coarse). The error estimates are distributed around the real error value when the mesh is sufficiently fine.

1. Introduction

Previous research suggests that Monte Carlo reactor physics simulations may benefit in various ways from usage of the fission matrix (Morton, 1956; Kalos et al., 1968; Carter and McCormick, 1969; Kadotani et al., 1991; Kitada and Takeda, 2001; Dufek and Gudowski, 2009a,b). The fission matrix is tallied during the actual Monte Carlo simulation using a spatial mesh superimposed over the system.

Although the fission matrix does contain statistical and other errors, the fundamental-mode eigenvector of the fission matrix is assumed to be a better representation of the fission source distribution than the actual Monte Carlo fission source. Various methods have therefore been suggested for improving the Monte Carlo fission source and accelerating its convergence using the fission matrix eigenvector (She et al., 2013; Urbatsch, 1995; Wenner and Haghighat, 2011; Carney et al., 2014; Pan et al., 2015; Nielsen et al., 2015).

Recently, Tuttelberg and Dufek (2014) showed that the fundamental-mode eigenvector of the fission matrix could also be used for estimating the error in the cumulative Monte Carlo fission source (i.e., the fission source combined over simulated cycles). The error in the cumulative Monte Carlo fission source can be used for deducing the error in the computed power distribution since the generated power and fission neutrons originate from the same fission sites. Knowledge of the error in computed power distributions is becoming essential with the recent

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development of coupled multi-physics simulations in which Monte Carlo solvers provide the computed power distribution to fuel depletion, thermal–hydraulic and other solvers.

As new tallies are added to the fission matrix at every criticality cycle, the statistical and other errors in the fission matrix decay, but they are always present in it. These errors may reflect themselves onto the fundamental-mode eigenvector, and introduce numerical instabilities into simulations that use the eigenvector for accelerating the fission source convergence (Dufek and Gudowski, 2009c). The errors in the fission matrix could also affect the accuracy of the eigenvector-based estimator of errors in the cumulative fission source. Tuttelberg and Dufek (2014) argue, however, that this estimator is not biased (i.e., it gives estimates around the real error) on the condition that errors in the eigenvector are not correlated to errors in the cumulative fission source.

The possible correlation of the errors in the eigenvector and the cumulative fission source has not been thoroughly studied in the previous works, although it may be of importance to all methods that work with the fission matrix. In this paper we study the possible correlation of the errors and its dependence on mesh coarseness. We complete this study with an analysis of the bias in the eigenvector-based estimator of errors in the cumulative fission source.

Methods that use the fundamental-mode eigenvector of the fission matrix are typically applied to all inactive as well as active cycles; i.e. the fission matrix starts to be tallied and used from the beginning of the Monte Carlo simulation. Therefore we do not distinguish between inactive and active cycles in our test simulations, and we mark all cycles as active for the purpose of the error correlation analysis.

Section 2 briefly describes the known theory of the fission matrix and the eigenvector-based error estimator. Results of several thousand numerical tests, covering error correlations and bias levels in the eigenvector-based error estimator are given in Section 3. Our conclusions are summarised in Section 4.

2. Theory

2.1. Errors in the cumulative fission source

The Monte Carlo fission source represents a collection of sites at which simulation of individual neutron histories begins. Similarly to other random variables, the fission source contains statistical errors of the order $O(1/\sqrt{m})$, where m is the number of neutron histories simulated per cycle (generation). The fission source also contains a systematic bias of the order $O(1/m)$ (Brissenden and Garlick, 1986). The statistical errors are caused by sampling the fission source at a limited number of fission sites using a random number generator, while the systematic bias in the fission source is caused by normalisation of the source to a required size at each cycle. Unlike the statistical errors, the bias does not decay over the cycles, and remains in the fission source during the whole simulation. Eventually, the fission source may also contain an error that propagated into it from the initial cycle if the source was sampled with a significant error there. While this error decays, over many cycles it may remain a significant part of the total error in systems with a large dominance ratio (Ueki et al., 2003).

At each cycle, new statistical errors propagate from the fission source into results – such as the multiplication factor and the power distribution. For this reason, results are always combined over a certain number, n, of cycles, which helps to reduce the statistical errors in results. Indeed, errors of other origins propagate from the fission source into the results as well; therefore, it can be inferred that errors in results are closely related to errors in the cumulative fission source (the fission source combined over the simulated cycles). Hence, knowledge of errors in the cumulative fission source can be very useful.

Let us define the relative scalar error in the cumulative fission source as

$$\varepsilon_{(\chi,n)} = \left\| \tilde{\vec{s}}_{(\chi,n)} - \tilde{\vec{z}}_{(\chi)} \right\|_1 \Big/ 2, \tag{1}$$

where $\vec{s}_{(\chi,n)}$ is the fission source combined over n cycles and discretised over a spatial mesh χ, $\vec{z}_{(\chi)}$ is the steady-state fission source discretised over the same mesh, and "$\sim$" denotes a normalisation operator defined for any vector $\vec{x}$ as

$$\tilde{\vec{x}} = \frac{\vec{x}}{\|\vec{x}\|_1}.$$

We apply the factor of 1/2 in Eq. (1) in order to limit the maximum value of ε to unity.

It is understood that the error ε, as defined by Eq. (1), is dependent on properties of the spatial mesh χ. The value of ε grows with refining the spatial mesh; making the mesh finer makes it possible to reflect more imperfections of the source into the discretised source vector. This is demonstrated in Section 3.6.

One could incorrectly assume that a very fine spatial mesh would allow to evaluate ε more accurately than a coarser mesh, as ε could then capture finer imperfections in the source. One needs to realise, however, that a sufficiently fine spatial mesh would always increase the value of ε to its maximum unity value, irrespective of the quality of the Monte Carlo fission source. This is because the Monte Carlo fission source is always sampled at a limited number of sites; hence, it is always possible to suggest a very fine mesh with so many zones that each zone would contain none or only a single fission neutron. In principle, it is therefore impossible to know the single correct value of the error in the fission source. Instead, we accept that the error is measured only when the source is discretised over a specific spatial mesh, and the mesh properties need to be specified together with the measured error. It follows from here that no "reference" ε value exists for a given fission source.

2.2. Eigenvector-based error estimator

The fundamental-mode fission source in Eq. (1) is unknown; therefore, Tuttelberg and Dufek (2014) suggest to substitute it by the fundamental-mode eigenvector of the fission matrix. The fission matrix **H** represents a space-discretised fission operator; the (i,j)th element of **H** gives the average number of neutrons born in a fission reaction in zone i, induced by a neutron born in zone j (Carter and McCormick, 1969). The fission matrix can be tallied over a number of cycles in a similar fashion as other results.

The error in the cumulative source is thus estimated as

$$\hat{\varepsilon}_{(\chi,n)} = \left\| \tilde{\vec{s}}_{(\chi,n)} - \tilde{\vec{h}}_{(\chi,n)} \right\|_1 \Big/ 2, \tag{2}$$

where $\vec{h}_{(\chi,n)}$ is the fundamental-mode eigenvector of the fission matrix that is tallied over the same cycles (and the same mesh χ) as the cumulative fission source $\vec{s}_{(\chi,n)}$. Note that Eq. (2) gives an estimation $\hat{\varepsilon}$ of ε.

Since the fission matrix contains errors, $\vec{h}_{(\chi,n)}$ cannot be identical to the correct fundamental-mode, and therefore $\hat{\varepsilon}$ must differ to ε. Nevertheless, Tuttelberg and Dufek (2014) show that $\hat{\varepsilon}$ is statistically distributed around ε if the errors in $\vec{s}_{(\chi,n)}$ and $\vec{h}_{(\chi,n)}$ are not correlated.

The correlation of errors in $\vec{s}_{(\chi,n)}$ and $\vec{h}_{(\chi,n)}$ is studied in Section 3. We are primarily interested in whether (and if so, then how) the possible correlation depends on the mesh coarseness. Dufek and