Reliability Engineering and System Safety 131 (2014) 298–304

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Nonparametric predictive inference for reliability of a k-out-of-m:G system with multiple component types

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ARTICLE INFO

Available online 4 May 2014

Keywords:
Diversity
Lower and upper probabilities
Nonparametric predictive inference
Redundancy
System reliability
k-out-of-m:G system

ABSTRACT

Nonparametric predictive inference for system reliability has recently been presented, with specific focus on k-out-of-m:G systems. The reliability of systems is quantified by lower and upper probabilities of system functioning, given binary test results on components, taking uncertainty about component functioning and indeterminacy due to limited test information explicitly into account. Thus far, systems considered were series configurations of subsystems, with each subsystem i a k_i-out-of-m_i:G system which consisted of only one type of components. Key results are briefly summarized in this paper, and as an important generalization new results are presented for a single k-out-of-m:G system consisting of components of multiple types. The important aspects of redundancy and diversity for such systems are discussed.

1. Introduction

Lower and upper probabilities generalize the standard theory of ('single-valued' or 'precise') probability and provide a powerful method for uncertainty quantification, see Utkin and Coolen [12] for an introductory overview from the perspective of reliability theory and applications and many further references. The main idea is that, for an event A, a lower probability $\underline{P}(A)$ and upper probability $\overline{P}(A)$ are specified, such that $0 \le \underline{P}(A) \le \overline{P}(A) \le 1$, with classical precise probability appearing in the special case with $\underline{P}(A) = \overline{P}(A)$. Like precise probability, lower and upper probabilities have different possible interpretations, including a subjective interpretation in terms of buying prices for gambles. Informally, a lower probability $\underline{P}(A)$ can be interpreted as reflecting the evidence in support of event A, which makes focus on lower probability for system functioning natural and attractive in reliability studies, we use this as the reliability measure of interest throughout this paper. For completeness, however, we also present the corresponding upper probability $\overline{P}(A)$, which can be interpreted by considering that $1-\overline{P}(A)$ reflects the evidence against event A, so in support of the complementary event A^c. The lower and upper probabilities presented in this paper are naturally linked by the conjugacy property $\overline{P}(A) = 1-\underline{P}(A^c)$ [1]. Zio [15] mentions the need for research into quantification of uncertainty in reliability by means of representations other than probability distributions. The lower and upper probabilities in NPI, as used in this paper, are optimal bounds on probabilities resulting from making only few assumptions. They enable statistical inference that can be considered 'objective' in a way that is not possible when restricting to classical probabilities [3].

Coolen [2] presented lower and upper predictive probabilities for Bernoulli random quantities, based on an assumed underlying latent variable model, with future outcomes of random quantities related to data by the assumption $A_{(n)}$ introduced by Hill [6]. These lower and upper probabilities are part of a wider statistical methodology called 'Nonparametric Predictive Inference' (NPI), which is a frequentist statistical approach with strong consistency properties [1], see Coolen [3] for an overview and further references (see also www.npi-statistics.com).

Coolen-Schrijner et al. [5] presented NPI for system reliability, in particular for series systems with subsystem i a k_i-out-of-m_i:G system. Such systems are common in practice, and can offer the important advantage of building in redundancy by increasing some m_i to increase the system reliability. Coolen-Schrijner et al. [5] considered the situation where each subsystem consists of components of a single type, with different subsystems having different types of components. They applied NPI for Bernoulli data to such systems, with inferences on each subsystem i based on information from tests on n_i components, where the tested components are assumed to be exchangeable with the corresponding components to be used in that subsystem. Only situations where components and the system either function or not when called upon were considered, we make the same assumption

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[1] Iain passed away in January 2012. We dedicate this paper, to which he fully contributed, to his memory.

http://dx.doi.org/10.1016/j.ress.2014.04.010
0951-8320/

throughout this paper. They presented an attractive algorithm for optimal redundancy allocation, with additional components added to subsystems one at a time, which in their setting was proven to be optimal. Hence, NPI for system reliability provides a tractable model, which greatly simplifies optimization problems involved with redundancy allocation. However, they only proved this result for tests in which no components failed. MacPhee et al. [9] generalized this result to redundancy allocation following tests in which any number of components can have failed, a situation in which redundancy plays possibly an even more important role than when testing revealed no failures at all.

Coolen et al. [4] considered NPI for system reliability in a similar setting, but with all subsystems consisting of the same single type of component. This is non-trivial, as the random quantities representing whether the components in the system function or not, are not independent in the NPI approach, given the test results. It is important that this dependence is explicitly taken into account, in particular when there is relatively little information from tests. We refer to Coolen-Schrijner et al. [5] for a discussion of this important aspect that seems often to be overlooked in reliability textbooks and applications, and which can lead to substantial mistakes in estimating system reliability.

This paper presents a further important step in the development of NPI for system reliability, where more general system structures can be considered. Whilst restricting attention to a single k-out-of-m:G system, this can now consist of multiple types of components. They are assumed to all play the same role within the system, but with regard to their reliability components of different types are assumed to be independent. The information from tests is also available per type of component.

Many recent contributions to the literature focus on reliability of systems related to those considered in this paper, albeit from a classical perspective using precise probabilities to quantify uncertainty. For example, Torres-Echeverria et al. [11] address modelling of probability of dangerous failure on demand and spurious trip rate of safety instrumented systems that include k-out-of-m voting redundancies in their architecture. Moghaddass et al. [10] consider a general repairable k-out-of-m:G system with non-identical components that can have different repair priorities. They address the problem of efficient evaluation of the system's availability in a way that steady state solutions can be obtained systematically with reasonable computation time. Vaurio [13] considers the unavailability of redundant standby systems with k-out-of-m logic. Such systems are subject to latent failures that are detected by periodic tests and repaired immediately after discovery. He considers many potential failure and error modes in the formalism, evaluates both consecutive and staggered testing schemes and he suggests methods for including common cause failures in the analyses. Further aspects of dependence of failures and the effect on system reliability are studied through simulations by Lin et al. [8]. Levitin [7] proposes a model that generalizes linear consecutive k-out-of-r-from-m systems to linear n-gap-consecutive k-out-of-r-from-m:F systems. He presents an algorithm for system reliability evaluation that is based on an extended universal moment generating function. Xing et al. [14] consider phased-mission requirements, with different numbers of components required to function at the different phases of the mission. These recent papers are evidence of the continuing importance of development of methodology to quantify system reliability, the NPI approach presented here provides the important opportunity to reflect, by the use of lower and upper probabilities, the fact that information from tests is often quite limited.

It is quite straightforward to combine the results presented in this paper with processes, considering reliability over time and based on test data consisting of failure times. One can just define as success the event that a component functions at age t, which then enables prediction of system reliability at a future time t, and this can be done for all t of interest to create nonparametric lower and upper survival functions for the system reliability. Although conceptually straightforward, this requires further research to enable important additional aspects to be included in the framework, including above-mentioned issues such as dependence between failures and phased missions. Furthermore, as testing is often required under severe time constraints, this provides excellent opportunities to combine this approach with methods and models for accelerated life testing, where the actual test information, typically from tests under increased levels of stress on the components, must be transformed to equivalent information reflecting the realistic stress levels prior to being embedded as data in the NPI approach in order to achieve the important exchangeability of failure times observed in testing and for the components in the system for which the reliability is to be predicted. As such an exchangeability assumption is necessarily subjective, so based on the judgements of experts with regard to the specific system, the usually subjective nature of the relevant judgement about the appropriateness of this assumption following transformation of data which are originating from accelerated life testing will form a natural component of the overall modelling assumptions that must be made in addition to, and not directly based on, available data. These topics make clear that there are many interesting research challenges and opportunities to further link the presented NPI approach with important practical aspects of system reliability.

Section 2 of this paper provides a brief introduction to NPI for system reliability. In Section 3 the main results of this paper are presented, namely the NPI lower and upper probabilities for functioning of a k-out-of-m:G system with multiple types of components. Section 4 presents examples to illustrate these lower and upper probabilities, and to discuss some specific related features including diversity. Section 5 concludes the paper with some remarks on planned further development of NPI for system reliability and related research challenges.

2. NPI for system reliability

Suppose that there is a sequence of $n+m$ exchangeable Bernoulli trials, each with 'success' and 'failure' as possible outcomes, and data consisting of s successes in n trials. Let Y_1^n denote the random number of successes in trials 1 to n, then a sufficient representation of the data for the inferences considered is $Y_1^n = s$, due to the assumed exchangeability of all trials. Let Y_{n+1}^{n+m} denote the random number of successes in trials $n+1$ to $n+m$. Let $R_t = \{r_1, \ldots, r_t\}$, with $1 \le t \le m+1$ and $0 \le r_1 < r_2 < \cdots < r_t \le m$. Then the NPI upper probability for the event $Y_{n+1}^{n+m} \in R_t$, given data $Y_1^n = s$, for $s \in \{0, \ldots, n\}$, is [2]

$$\overline{P}(Y_{n+1}^{n+m} \in R_t | Y_1^n = s) = \binom{n+m}{n}^{-1} \sum_{j=1}^{t} \Delta C_j \binom{n-s+m-r_j}{n-s}$$

with $\Delta C_1 = \binom{s+r_1}{s}$ and $\Delta C_j = \binom{s+r_j}{s} - \binom{s+r_{j-1}}{s}$, $2 \le j \le t$. The corresponding NPI lower probability is derived via the conjugacy property $\underline{P}(Y_{n+1}^{n+m} \in R_t | Y_1^n = s) = 1 - \overline{P}(Y_{n+1}^{n+m} \in R_t^c | Y_1^n = s)$, where $R_t^c = \{0, 1, \ldots, m\} \backslash R_t$. Coolen [2] derived these NPI lower and upper probabilities through direct counting arguments. The method uses the appropriate $A_{(n)}$ assumptions [6] for inference on m future random quantities given n observations, and a latent variable representation with Bernoulli quantities represented by observations on the real-line, with a threshold such that successes are to one side and failures to the other side of the threshold.

Under these assumptions, the $\binom{n+m}{n}$ different orderings of these observations, when not distinguishing between the n observed values nor between the m future observations, are all equally