New aspects in the implementation of the quasi-static method for the solution of neutron diffusion problems in the framework of a nodal method

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ABSTRACT

The ability to accurately model the dynamic behaviour of the neutron distribution in a nuclear system is a fundamental aspect of reactor design and safety assessment. Due to the heavy computational burden associated to the direct time inversion of the full model, the quasi-static method has become a standard approach to the numerical solution of the nuclear reactor dynamic equations on the full phase space. The present paper is opened by an introductory critical review of the basics of the quasi-static scheme for the general neutron kinetic problem. Afterwards, the implementation of the quasi-static method in the context of a three-dimensional nodal diffusion theory model in hexagonal-*z* geometry is described, including some peculiar aspects of the adjoint nodal equations and the explicit formulation of the quasi-static nodal equations. The presentation includes the discussion of different formulations of the quasi-static technique. The results presented illustrate the features of the various formulations, highlighting the corresponding advantages and drawbacks. An adaptive procedure for the selection of the time interval between shape recalculations is also presented, showing its usefulness in practical applications.

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1. Introduction

The design and safety assessment of the innovative nuclear systems proposed for future development requires the accurate simulation of the behaviour of the neutron distribution during typical operational and accidental conditions. As the time inversion of the full model is often impractical due to computational limitations or even unnecessary as a result of the characteristics of the physical processes, the quasi-static method initially proposed long ago (Henry, 1958) has emerged as a standard approach to the modelling of nuclear reactor dynamics on the full phase space.

The present work is oriented towards the implementation of the quasi-static method in a nodal diffusion theory solver which comprises the neutronic module of a coupled neutronics/thermal-hydraulics reactor analysis code (Bonifetto et al., 2013b; Bonifetto et al., 2013a). Following a general review of the theoretical basis of the quasi-static method and solution algorithms, a detailed description of the implementation of various formulations of the method in the framework of a nodal discretisation of the time-dependent multigroup diffusion

equations is provided. The results presented demonstrate the behaviour, both in terms of accuracy and performance, of different variants of the quasi-static method with respect to the reference, full inversion method.

2. Theoretical basis of the quasi-static method

The theoretical basis of the quasi-static method of solution of the time-dependent neutronics problems is critically reviewed in the context of neutron transport theory. The problem is first posed in the framework of the proper mathematical equations which describe the relevant physical phenomena. Afterwards, the fundamental hypotheses of the quasi-static approach are stated, thereby permitting the derivation of the full system of kinetics equations to be solved by the quasi-static algorithm. Finally, the available methods of solution of these equations are discussed.

2.1. Problem setting

The time-dependent neutron transport equation and delayed neutron precursors balance equations for a stationary medium may be written as

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$$\begin{cases} \dfrac{1}{v(E)}\dfrac{\partial}{\partial t}\phi(\mathbf{r},E,\mathbf{\Omega},t)=-\mathcal{L}(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t)+\mathcal{M}_p(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t) \\ \quad +\sum\limits_{i=1}^{R}\dfrac{\chi_i(\mathbf{r},E)}{4\pi}\lambda_i c_i(\mathbf{r},t)+S(\mathbf{r},E,\mathbf{\Omega},t), \\ \dfrac{\chi_i(\mathbf{r},E)}{4\pi}\dfrac{\partial}{\partial t}c_i(\mathbf{r},t)=\mathcal{M}_i(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t)-\dfrac{\chi_i(\mathbf{r},E)}{4\pi}\lambda_i c_i(\mathbf{r},t), \quad i=1,\dots,R, \end{cases}$$

$$(1)$$

and subject to appropriate initial and boundary conditions. In Eq. (1), $\phi(\mathbf{r},E,\mathbf{\Omega},t)$ represents the time-dependent angular neutron flux, $c_i(\mathbf{r},t)$ is the time-dependent delayed neutron precursor concentration for delayed neutron precursor family i and the operators are defined by

$$\mathcal{L}(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t)\equiv\mathbf{\Omega}\cdot\nabla\phi(\mathbf{r},E,\mathbf{\Omega},t)+\Sigma_t(\mathbf{r},E,t)\phi(\mathbf{r},E,\mathbf{\Omega},t)$$
$$-\int dE'\oint d\Omega'\,\Sigma_s(\mathbf{r},E'\to E,\mathbf{\Omega}'\cdot\mathbf{\Omega},t)\phi(\mathbf{r},E',\mathbf{\Omega}',t),$$

$$(2a)$$

$$\mathcal{M}_p(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t)\equiv(1-\beta)\dfrac{\chi_p(\mathbf{r},E)}{4\pi}$$
$$\times\int dE'\oint d\Omega'\,v\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',\mathbf{\Omega}',t),\quad (2b)$$

$$\mathcal{M}_i(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t)\equiv\beta_i\dfrac{\chi_i(\mathbf{r},E)}{4\pi}$$
$$\times\int dE'\oint d\Omega'\,v\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',\mathbf{\Omega}',t).\quad (2c)$$

In these expressions, the operator $\mathcal{L}(\mathbf{r},E,\mathbf{\Omega},t)$ is the total reaction operator except for fission and $\mathcal{M}_p(\mathbf{r},E,\mathbf{\Omega},t)$ and $\mathcal{M}_i(\mathbf{r},E,\mathbf{\Omega},t)$ are the prompt and delayed fission operators, respectively. Similarly, the total fission operator is defined as

$$\mathcal{M}(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t)\equiv\mathcal{M}_p(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t)$$
$$+\sum_{i=1}^{R}\mathcal{M}_i(\mathbf{r},E,\mathbf{\Omega},t)\phi(\mathbf{r},E,\mathbf{\Omega},t).\quad (3)$$

A reference system is defined as the initial, unperturbed steady-state configuration from which the transient is initiated. By taking all time derivatives in Eq. (1) to be zero and by assuming that the delayed neutron precursors concentrations are in equilibrium with the steady-state neutron flux, the static neutron transport equation is obtained

$$-\mathcal{L}_0(\mathbf{r},E,\mathbf{\Omega})\phi_0(\mathbf{r},E,\mathbf{\Omega})+\mathcal{M}_0(\mathbf{r},E,\mathbf{\Omega})\phi_0(\mathbf{r},E,\mathbf{\Omega})+S_0(\mathbf{r},E,\mathbf{\Omega})=0,$$

$$(4)$$

where the subscript indicates the static, reference system. In addition to the solution of the direct equation of the reference system, it is possible to obtain the solution of the corresponding adjoint equation, the solution of which is physically significant as it represents the distribution of the neutron importance (Wigner, 1992; Ussachoff, 1956; Lewins, 1965). The adjoint of Eq. (4) is

$$-\mathcal{L}_0^\dagger(\mathbf{r},E,\mathbf{\Omega})\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega})+\mathcal{M}_0^\dagger(\mathbf{r},E,\mathbf{\Omega})\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega})+S_0^\dagger(\mathbf{r},E,\mathbf{\Omega})=0,$$

$$(5)$$

where the dagger $\dagger$ represents an adjoint operator or an adjoint quantity. Both Eqs. (4) and (5) are accompanied by direct and adjoint boundary conditions, respectively.

2.2. The quasi-static method

The quasi-static method of solution of the time-dependent neutron balance equation and delayed neutron precursors balance equations (Henry, 1958; Ott and Meneley, 1969; Devooght and Mund, 1980) is based on the rationale that the neutron flux may

be factorised into the product of two separate but correlated functions of the form

$$\phi(\mathbf{r},E,\mathbf{\Omega},t)\equiv T(t)\psi(\mathbf{r},E,\mathbf{\Omega},t),\quad (6)$$

where the amplitude function $T(t)$ depends only on time and the shape function $\psi(\mathbf{r},E,\mathbf{\Omega},t)$ depends on all of the phase space variables. The amplitude function, which is proportional to the magnitude of the total reactor power, is intended to follow the fastest evolving time scales while the shape function accounts for spatial and spectral variations whose temporal evolution occurs on a slower time scale. This assumption, combined with a suitable projection procedure, allows to separate the original system of partial differential equations for the neutron flux and the delayed neutron precursors concentrations, Eq. (1), into two coupled systems of differential equations: a coupled set of ordinary differential equations for the amplitude and a coupled set of partial differential equations for the shape. The advantage of these operations is that it becomes possible to solve separately for the unknowns on their respective time scales, potentially offering a significant reduction of computational effort based on the nature of the transient and the degree to which the system is coupled.

The mathematical formalism for the projection-separation procedure is as follows. The flux factorisation, Eq. (6), is rendered unique by imposing an initial condition on the amplitude function, $T(0)=1$, and a normalisation condition (Henry, 1958) on the shape function

$$\left\langle\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),\frac{1}{v(E)}\psi(\mathbf{r},E,\mathbf{\Omega},t)\right\rangle=\gamma(t)=\gamma_0,\qquad\forall t,\quad (7)$$

with γ_0 an arbitrary constant and the scalar product notation $\langle\cdot,\cdot\rangle$ denoting the integration over all independent phase space variables.

Substitution of the factorised form of the flux into Eq. (1), projection of both onto the adjoint solution of the reference system, subtraction of Eq. (5) projected onto the solution of the direct problem from the first of the projected Eq. (1) and the use of the normalisation condition allows to write the amplitude equations

$$\begin{cases} \dfrac{d}{dt}T(t)=\dfrac{\rho(t)-\widetilde{\beta}(t)}{\Lambda(t)}T(t)+\sum\limits_{i=1}^{R}\lambda_i\widetilde{c}_i(t)+\widetilde{s}(t), \\ \dfrac{d}{dt}\widetilde{c}_i(t)=\dfrac{\widetilde{\beta}_i(t)}{\Lambda(t)}T(t)-\lambda_i\widetilde{c}_i(t),\quad i=1,\dots,R, \end{cases}$$

$$(8)$$

with the integral kinetics parameters given by the definitions

$$F(t)\equiv\langle\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),\mathcal{M}(\mathbf{r},E,\mathbf{\Omega},t)\psi(\mathbf{r},E,\mathbf{\Omega},t)\rangle,\quad (9a)$$

$$\Lambda(t)\equiv\frac{1}{F(t)}\left\langle\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),\frac{1}{v(E)}\psi(\mathbf{r},E,\mathbf{\Omega},t)\right\rangle,\quad (9b)$$

$$\widetilde{\beta}_i(t)\equiv\frac{1}{F(t)}\langle\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),\mathcal{M}_i(\mathbf{r},E,\mathbf{\Omega},t)\psi(\mathbf{r},E,\mathbf{\Omega},t)\rangle,\quad (9c)$$

$$\rho(t)\equiv\frac{1}{F(t)}\langle\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),[-\delta\mathcal{L}(\mathbf{r},E,\mathbf{\Omega},t)+\delta\mathcal{M}(\mathbf{r},E,\mathbf{\Omega},t)]\psi(\mathbf{r},E,\mathbf{\Omega},t)\rangle$$
$$-\frac{1}{F(t)}\left\langle S_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),\psi(\mathbf{r},E,\mathbf{\Omega},t)\right\rangle,\quad (9d)$$

$$\widetilde{c}_i(t)\equiv\frac{1}{\Lambda(t)F(t)}\left\langle\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),\frac{\chi_i(\mathbf{r},E)}{4\pi}c_i(\mathbf{r},t)\right\rangle,\quad (9e)$$

$$\widetilde{s}(t)\equiv\frac{1}{\Lambda(t)F(t)}\langle\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}),S(\mathbf{r},E,\mathbf{\Omega},t)\rangle,\quad (9f)$$

in which the delta notation δ indicates the absolute perturbation of an operator with respect to its value for the reference system.