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# Dynamic availability assessment and optimal component design of multi-state weighted k-out-of-n systems

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ABSTRACT

Availability/reliability is a main feature of design and operation of all engineering systems. Recently, availability evaluation of multi-state systems with different structures is at the center of attention due to the wide applications in engineering. In this paper, a dynamic model is developed for the availability assessment of multi-state weighted k-out-of-n systems. Then, in a design optimization problem, the availability and capacity for the components of such systems are optimized by genetic algorithm. In the dynamic model, the probabilities and capacities of components in different states are allowed to be changed over time. For availability assessment, universal generating function and Markov process are adopted. Application of the proposed model is illustrated using a real-world marine transportation system in order to evaluate and compare the presented optimization problems in assessing system availability.



## 1. Introduction

Weighted multi-state systems are composed of multi-state components which have different performance levels and several failure modes, and such a model is much more realistic in studying system reliability [1–4]. A multi-state system may have a basic architecture such as series, parallel, k-out-of-n, and network. The k-out-of-n structure is a very popular structure of the multi-state systems with wide application and research works [5–10]. The multi-state weighted k-out-of-n system is a type of multi-state systems that has wide spread applications such as in traffic systems, telecommunication networks, and satellites [11]. Due to the importance and wide application of multi-state systems, many research works have been devoted to model the availability/reliability of these systems.

Li and Zuo [2] reviewed the methods for availability or reliability assessment of multi-state systems, and applied a recursive algorithm for availability assessment of multi-state weighted k-out-of-n system in a non-dynamic model. Most reliability/-availability studies of multi-state weighted k-out-of-n system pre-assumed that the state probability of system/component does not change throughout system lifetime. However, complex systems are often subjected to aging process which implies that the system/component state probability may gradually change with time [12]. Therefore, it is of large practical value to model the state probability as a function of time. In the recursive algorithm used by Li and Zuo [2], the formula should be broken down into sub-formulas for calculation of reliability. When the factor "time" is considered in the sub-formulas, the method becomes more complex and computational. In this case, the system availability can be obtained using Universal Generating Function (UGF) [13]. UGF application can deal with the dynamic reliability evaluation.

Mostly, UGF has been used for reliability calculation of multi-state systems with different types of structures such as series-parallel and bridge [14–19]. Application of UGF was for these types rather than multi-state k-out-of-n systems.

Liu and Kapur [20] developed reliability measures and analyzed reliability for dynamic non-repairable multistate systems. As systems become more complex, achieving an optimal design has been of great importance in recent years. The design of a system is often evaluated by four types of measures: reliability, availability, mean time to failure, and percentile life [21]. In this paper, by availability evaluation, we investigate the optimal design of a dynamic multi-state system.

Li and Zuo [11] presented a study on reliability assessment and optimal design of multi-state weighted k-out-of-n systems for a non-dynamic model. In their work, the objective was to select the component choices to minimize the system cost subject to requirement on system availability. In this paper, we modify the objective function presented by Li and Zuo [11] to minimize the cost and find the optimal system design in dynamic model. The rest of this paper is organized as follows. Section 2 presents a dynamic model to assess the availability of multi-state weighted k-out-of-n: G systems. In Section 3, a dynamic design problem is introduced to be solved by genetic algorithm. In Section 4, one real-world example from

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**Nomenclature**

| | |
|---|---|
| $N$ | The number of components of the system. |
| $M$ | The best operating state for the components of the system, $M+1$ : total number of states. |
| $i$ | Index of component number in the system $1 \le i \le N$. |
| $j$ | Index of component state in the system $0 \le j \le M$. |
| $c_i$ | The design and manufacturing cost of component $i$. |
| $k_j$ | The minimum total capacity required to ensure that the system is in state $j$ or above. |
| $C_j$ | Cost of system being in state below $j$ (cost of failure). |
| $A(t)$ | Availability of a multi-state k-out-of-n system at time $t$. |
| $k(t)$ | The demand capacity to ensure that the system is working properly at time $t$. |
| $u_{ij}$ | Capacity of component $i$ in state $j$. |
| $u_{ij}(t)$ | Capacity of component $i$ in state $j$ at time $t$. |
| $p_{ij}$ | Probability of component $i$ being in state $j$. |
| $p_{ij}(t)$ | Probability of component $i$ being in state $j$ at time $t$. |
| $\lambda^i_{j,k}$ | Transition (failure) rate of component $i$ from state $j$ to state $k$ $(j>k)$. |
| $\mu^i_{jk}$ | Transition (repair) rate of component $i$ from state $j$ to state $k$ $(j<k)$. |
| $\Phi$ | The system's structure function representing the state of the system. |
| $K$ | Total capacity of all components of the system. |
| $\hat{A}_{Sys}$ | The minimum required probability for the system to attain a state of $j$ or above. |
| $C_{Sys}$ | Total cost of the system. |

maritime transportation system is used to apply the dynamic availability model. Conclusions are provided in Section 5.

## 2. Dynamic availability model

In weighted k-out-of-n: G systems, each component of the system and the whole system have $(M+1)$ states: 0, 1, 2… $M$. In Fig. 1, a general Markov model for a system with $N$ components and with $(M+1)$ states is presented. Component $i(1 \le i \le N)$ in state $j(0 \le j \le M)$ has a capacity value of $u_{ij}$. System is in state $j$ or above if the total capacity of all components is larger than or equal to the value $k_j$. Then, this definition means

$$Pr\{\Phi \ge j\} = Pr\{K \ge k_j\} \tag{1}$$

In dynamic availability assessment of multi-state weighted k-out-of-n systems, we consider a time function for probability distribution of component $i$ in state $j$ as $p_{ij}(t)$. The probability functions $p_{ij}(t)$ of the components are obtained from Chapman–Kolmogorov Eqs. (2)–(4). Then, the system probability function is obtained from system's universal generating function (UGF).

$$\begin{cases} \frac{dp_{i0}(t)}{dt} = \sum\limits_{j=1}^{M} \lambda^i_{j0} p_{ij}(t) - \sum\limits_{j=1}^{M} p_{ij}(t)\mu^i_{0j} \\ \vdots \\ \frac{dp_{ik}(t)}{dt} = \sum\limits_{j=k+1}^{M} \lambda^i_{jk} p_{ij}(t) + \sum\limits_{j=0}^{k-1} p_{ij}(t)\mu^i_{jk} - p_{ik}(t)\left(\sum\limits_{j=0}^{k-1} \lambda^i_{kj} + \sum\limits_{j=k+1}^{M} \mu^i_{kj}\right) \\ \vdots \\ \frac{dp_{iM}(t)}{dt} = \sum\limits_{j=0}^{M-1} \mu^i_{jM} p_{ij}(t) - \sum\limits_{j=1}^{M} p_{iM}(t)\lambda^i_{Mj} \end{cases} \tag{2}$$

According to the definition of the system, the sum of all state probabilities at any time should be equal to 1. That is,

$$\sum_{j=0}^{M} p_{ij}(t) = 1 \tag{3}$$

The Eqs. (2) and (3) can be solved simultaneously, with the initial conditions

$$p_{iM}(0) = 1,\ \ldots,\ p_{ik}(0) = 0,\ \ldots,\ p_{i0}(0) = 0, \tag{4}$$

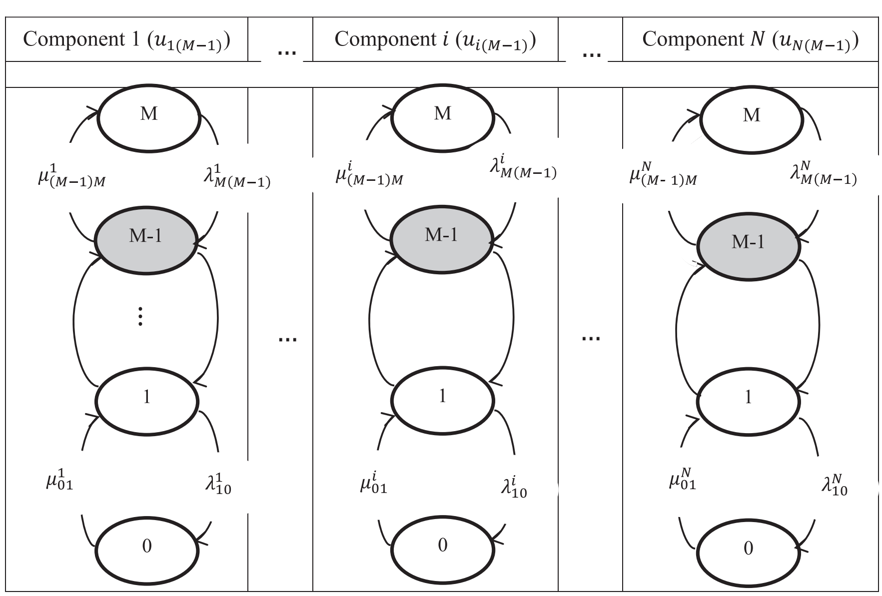


**Fig. 1.** A general Markov model for a system with $N$ components, e.g. if all components are in state $(M-1)$, then the system is in state $j$ or above if $k_j = N(M-1)$, $u_{i(M-1)} = M-1 \quad for\ \forall i \in \{1,\ldots,N\}$.