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Incorporating risk seeking attitude into defense strategy

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ABSTRACT

Optimal resource allocation is first found in defending possible targets against random terrorist attacks subject to budget constraint. The mathematical model is a nonconvex optimization problem which can be transformed into a convex problem by introducing new decision variables, so standard methods can be used for its solution. Without budget constraint the simplified model can be solved by a very simple algorithm which requires the solution of a single variable monotone equation.

1. Introduction

Defending objectives, which can be the targets of terrorist attacks, is one of the most important goals of homeland security. The outcomes of the actions of the defender are uncertain because they also depend on the random actions of the attacker. Game theory is the most appropriate approach to model the interactions between the attacker and the defender. Attacker–defender games have been intensively studied in recent years. Some researchers consider the players' payoffs as deterministic values and assume that the defender seeks to minimize the damage, while the attacker tries to maximize it [5,7]. However, the players' payoffs are usually random due to the uncertainty in the game, and therefore classic equilibrium approach has its limitations to find the solutions under this situation. Risk analysis is often used to capture the uncertainty resulting by the presence of random variables in the players' payoff functions. A production and conflict model is introduced and analyzed in [6] when two agents are fighting for as large as possible shares of the total production, which is determined by their contest success function. A two-person conflict model is discussed in [22] when the agents can select between converting resources into arms or into useful production. The wining probabilities of the agents depend on their armament levels and the obtained reward depends on the amount of the useful production. The Nash equilibrium of this two-person game is determined and its dependence on the risk-taking attitudes of the agents is examined. A new contest model is introduced in [4] which is an extension and generalization of the rent-seeking games where contest functions determine the winning probabilities and exponential utility function are assumed. The existence and uniqueness of the equilibrium is proved in the special case when every player has a constant degree of absolute risk aversion. Comparative static results are proved showing how the utility dissipation is affected by the risk-taking attitude of the agents and the precise nature of the technology. The central moments describe the nature of the distribution of a random variable mathematically, and any distribution can be characterized by the mean, the variance, the skewness, etc [19]. The first moment is usually considered as the payoffs of the players in attacker–defender games. For instance, Bier et al. [2] develop optimal strategies to allocate resources among possible defensive investments based on the assumptions that the attacker and the defender will maximize and minimize the expected damage of an attack on the system, respectively. In order to find best strategic defensive allocation against an unknown attacker, Bier et al. [3] consider cases when the attacker seeks to maximize the expected payoff from launching an attack and the defender tries to minimize the expected loss of an attack. Hausken and Zhuang [11,12] employ contest success function to describe the probability of damage and compute the government's and terrorists' expected utilities in the attacker–defender game. Azaiez and Bier [1] claim that the defender maximizes the expected cost of the attack by considering an investment to strengthen the defense capability of the object. Levitin [13] suggests optimal defense strategy that presumes separation and protection of system elements based on the consideration that the attacker tries to maximize the expected damage of an attack. Levitin and Hausken [14] propose that the defender can enhance system reliability by

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either protecting a subset of the genuine system elements or deploying separated redundant elements and false elements. The agents' optimal strategies can be obtained from the expected damage when the cumulative performance of the system elements cannot meet a demand. Hausken and Levitin [8,9] consider the interaction of the attacker and the defender as a two-person simultaneous game where damages might occur to several elements of the system according to a binomial distribution, and the possible damage is determined with a general contest function including the intensity of the contest. Wang et al. [24] discuss the attacker–defender problem and analyze how to allocate resources to maximize the probability of core services availability by considering all kinds of components in the entire system. Since the values of the core services can be estimated from the demand, the objective function of the defender is equivalent to the expected value of the available services under the attacks. Similarly, the expected payoffs to the attacker and to the defender are also employed by several scholars in conducting research on systems defense and attack models [25,26,18,20]. To our best knowledge, in all earlier studies expected value of the random objective is considered and optimized [10].

Clearly, the characterization of a random variable becomes more accurate if higher moments are also considered. However, the complexity of the computation increases as well. Modeling the uncertainties in this game should include the risk seeking attitude of the players. As it is common in the economic literature, uncertain outcomes are substituted with their certainty equivalents [21] including the first two central moments of the random variables. The certainty equivalent of the payoff of the defender is a linear combination of its expectation and variance, where a risk attitude coefficient is assigned to the variance.

In this paper it is assumed that there are several possible targets which can be attacked, and the defender has the assessment of the probability for each possible target to be attacked. These probability values can be obtained by using actual data from previous interactions with the attacker, or from information about its capabilities or from other sources. The question is to find optimal resource allocation strategy of the defender prior to the attack. This paper offers a mathematical model and solution algorithm for this problem before an actual attack occurs. Instead of computing the amount of the damage by a contest function, we determine the proportion of the maximum possible damage which can be avoided by the protecting actions of the defender. For the sake of mathematical simplicity we use a simple form of the contest function, more general forms (such as used in [6]) including intensity can be applied in a similar way. We also incorporate risk by including the variance of the random payoff of the defender into the objective function. After an attack occurs, the defender responds to it, then newer attack occurs, the defender responds again, and so on. A possible model and solution procedure are offered for the resulting multistage stochastic game for example, in our earlier work [16] and in the other papers mentioned earlier.

In developing the mathematical model, we will first derive the payoff function of the defender including the random elements, and then its certainty equivalent will be determined based on its expectation and variance. However, this payoff function is not concave in general, so standard methodology cannot be used. By introducing new decision variables both the objective function and the budget constraint become concave, so the model is transformed into a convex programming problem.

The rest of the paper is organized as follows. Section 2 introduces the mathematical model and the transformation into a convex programming problem is given in Section 3. In the case of unlimited or very high available budget an unconstrained optimization problem is obtained, its special solution algorithm is introduced in Section 4. An illustrative example is given in Section 5. The last Section 6 concludes the paper with future research directions.

2. The mathematical model

Suppose there are I independent possible targets, and let i be the index of them ($i=1, 2,\dots, I$). The intruder is assumed to attack one target at each time. Combined attacks can be considered as single attacks, since we can consider the combinations of targets as new targets, and so combined attacks as separate attacks. Let n_i ($i=1, 2,\dots, I$) be the effort of the attacker to attack target i and let p_i ($i=1, 2,\dots, I$) be the probability of the actual attack. Let v_i ($i=1, 2, \dots, I$) be the highest possible damage in object i if it is attacked and the object is unprotected, and let m_i ($i=1, 2,\dots, I$) be the effort of the defender to protect target i against the attack. In addition, let c_i ($i=1, 2,\dots, I$) be the unit cost of this effort. It is assumed that the defender is able to avoid $m_i/(m_i+n_i)$ proportion of the possible highest damage if it is actually attacked. This simple formula can be replaced by more sophisticated expressions, which would not significantly modify the model. This expression is very similar to the contest function concept from economics. So in this case the defender's payoff is $z_i=v_i\, m_i/(m_i+n_i)-c_i m_i$, which occurs with probability p_i. Here we assume that defending action is made only in the case of an attack. If a possible target is not attacked, then the defending resources remain idle there. The defender's payoff is a discrete random variable with possible values z_i and occurring probabilities p_i ($i=1, 2,\dots, I$). Therefore the expectation and variance of the defender's payoff z are as follows:

$$E(z)=\sum_{i=1}^{I}\left(v_i\frac{m_i}{m_i+n_i}-c_i m_i\right)p_i \quad \textit{and}$$
$$Var(z)=\sum_{i=1}^{I}\left(v_i\frac{m_i}{m_i+n_i}-c_i m_i\right)^2 p_i-(E(u))^2 \tag{1}$$

If r denotes the risk seeking attitude of the defender, then the certainty equivalent ([21]) of its random payoff is given as

$$D=\sum_{i=1}^{I}\left(v_i\frac{m_i}{m_i+n_i}-c_i m_i\right)p_i-r\sum_{i=1}^{I}\left(v_i\frac{m_i}{m_i+n_i}-c_i m_i\right)^2$$
$$p_i+r\left[\sum_{i=1}^{I}\left(v_i\frac{m_i}{m_i+n_i}-c_i m_i\right)p_i\right]^2. \tag{2}$$

The value $r=0$ refers to risk neutral attitude, $r>0$ to risk aversion and $r<0$ to risk seeking behavior. In this study, we assume that $r\geq 0$, that is, the risk seeking behavior is excluded, which is not realistic in our case, as it will be explained later. The decision variables are m_i ($i=1, 2,\dots, I$) and the values of v_i, n_i, c_i, p_i ($i=1, 2,\dots, I$) and r are assumed to be known by the defender. Let B denote the defender's available budget, then the budget constraint can be formulated as

$$\sum_{i=1}^{I} m_i \leq B. \tag{3}$$

Hence our model is to maximize the objective function (2) subject to the budget constraint (3). Notice that the objective function (2) can be interpreted as using the weighting method in a multiobjective programming problem, where the first objective is to maximize expectation and the second objective is to minimize variance.

Notice that constraint (3) is linear, however the objective function (2) is non-concave in general, so standard iteration methods [15] cannot be used to find optimum. In the next section