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How coupon and element tests reduce conservativeness in element failure prediction

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ABSTRACT

Structural elements, such as stiffened panels, are designed by combining material strength data obtained from coupon tests with a failure theory for 3D stress field. Material variability is captured by dozens of coupon tests, but there remains epistemic uncertainty due to error in the failure theory, which can be reduced by element tests. Conservativeness to compensate for the uncertainty in failure prediction (as in the A- or B-basis allowables) results in a weight penalty. A key question, addressed here, is what weight penalty is associated with this conservativeness and how much it can be reduced by using coupon and element tests. In this paper, a probabilistic approach is used to estimate the conservative element failure strength by quantifying uncertainty in the element strength prediction. A convolution integral is used to efficiently combine uncertainty from coupon tests and that from the failure theory. Bayesian inference is then employed to reduce the epistemic uncertainty using element test results. The methodology is examined with typical values of material variability (7%), element test variability (3%), and the error in the failure theory (5%). It is found that the weight penalty associated with no element test is significant (20% heavier than an infinite number of element tests), and it is greatly reduced by more element tests (4.5% for 5 element tests), but the effect of the number of coupon tests is much smaller.

1. Introduction

Uncertainty has always been a major concern in structural design. For example, predicting the strength of a structural element has two major sources of epistemic uncertainty (uncertainty associated with the lack of information). The first comes from errors in failure prediction based on calculated stresses and a failure theory. The second source is errors in measuring variability of material properties. Coupon tests are performed to measure material variability, but the estimated variability has error due to the limited number of coupons.

Aircraft designers use conservative measures, such as A- or B-basis allowable, to compensate for uncertainty in material strength prediction as in MIL-HDBK [1]. For example, the B-basis introduces conservativeness in two ways. To compensate for variability, the B-basis uses the lower 10% value of the material strength distribution. However, calculating the lower 10% relies on the number of coupons, which brings in epistemic uncertainty. Thus, the B-basis requires an additional 95% confidence level to compensate for the epistemic uncertainty. That is, the B-basis provides a value that belongs to the lower 10% with 95% probability. The B-basis is calculated based on a sample mean and standard deviation with a factor for one-sided tolerance limit with an assumed population distribution. MIL-HDBK [1] and Owen et al. [2] presented tables of the factors with various population distributions. To compensate for the error in a failure theory, it is common practice to repeat element tests three times and then select the lowest test result as a conservative estimate of the failure envelope; this process can be interpreted as applying a knockdown factor on the average test result.

Treating epistemic uncertainty is reflected in the literature of probabilistic design. Noh et al. [3] compensated for epistemic uncertainty caused by the finite number of samples with a confidence level of 97.5%. Matsumura et al. [4] and Villanueva et al. [5] considered the effect of epistemic uncertainty in a computer model on estimating probability of failure of an integrated thermal protection system of a space vehicle and demanded 95% confidence for the epistemic uncertainty.

These conservative statistical approaches have worked successfully to achieve the safety of structural designs. However, they were applied at an individual test stage without considering their overall efficiency to achieve the safety level at the final stage. Also, it has not been quantified how much these tests reduce the weight penalty compared to the design without tests.

When we use failure theory to predict the strength of an element, we propagate uncertainty in coupons and combine it with uncertainty in the failure theory. We build and test the

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Nomenclature

b_e error bound for failure theory
b_σ estimated bound for standard deviation of structural element
$\hat{e}_{k,Ptrue}$ possible true error in failure theory
$f^{init}(\mu_{e,Ptrue}, \sigma_{e,Ptrue})$ initial joint PDF for given mean and standard deviation of structural element
$f_{k,Ptrue}(e_{k,Ptrue})$ PDF for given possible true error in failure theory
$f_{\mu c,Ptrue}(\mu_{c,Ptrue})$ PDF for given possible true mean of material strength
$f_{\mu e,Ptrue}(\mu_{e,Ptrue})$ PDF for given possible true mean of structural strength
$f_{\sigma c,Ptrue}(\sigma_{c,Ptrue})$ PDF for given possible true standard deviation of material strength
$f_{\sigma e,Ptrue}(\sigma_{e,Ptrue})$ PDF for given possible true standard deviation of structural strength
$f^{upd}(\mu_{e,Ptrue}, \sigma_{e,Ptrue})$ updated joint PDF for given mean and standard deviation of structural element
$f^{upd}_{\mu e,Ptrue}(\mu_{e,Ptrue})$ updated marginal distribution for given mean of structural element
$f^{upd}_{\sigma e,Ptrue}(\sigma_{e,Ptrue})$ updated marginal distribution for given standard deviation of structural element
$k_{3d,calc}$ calculated ratio of structural element strength to material strength
$\hat{k}_{3d,Ptrue}$ possible true structural element strength to material strength
$k_{3d,true}$ true ratio of structural element strength to material strength
$l^i_{test}(\mu_{e,Ptrue}, \sigma_{e,Ptrue})$ likelihood function of ith test for given mean and standard deviation of structural element
$\mu_{0.05}$ mean of 5th percentile of the mean element strength for given test result
$\hat{\mu}_{c,Ptrue}$ possible true mean of material strength
$\mu_{c,test}$ measured mean of material strength from coupon test
$\mu_{c,true}$ true mean of material strength
$\hat{\mu}_{e,Ptrue}$ possible true mean of structural element strength
$\mu_{e,test}$ measured mean of structural element strength from coupon test
$\mu_{e,true}$ true mean of structural element strength
n_c the number of coupon tests
n_e the number of element tests
PUD probability of unconservative design
PTD possible true distribution
$\hat{\sigma}_{c,Ptrue}$ possible true standard deviation of material strength
$\sigma_{c,test}$ measured standard deviation of material strength from coupon test
$\sigma_{c,true}$ true standard deviation of material strength
$\hat{\sigma}_{e,Ptrue}$ possible true standard deviation of structural element strength
$\sigma_{e,test}$ measured standard deviation of structural element strength from coupon test
$\sigma_{e,true}$ true standard deviation of structural element strength
$\tau_{0.05}$ 5th percentile of the mean element strength for given test results
$\hat{\tau}_{c,Ptrue}$ possible true material strength
$\hat{\tau}_{c,true}$ true material strength
$\hat{\tau}_{e,Ptrue}$ possible true structural element strength
$\hat{\tau}_{e,true}$ true structural element strength
$w_{0.95}$ 95th percentile of the weight penalty for given test results

Superscripts

init initial distribution (prior distribution)
upd updated distribution (posterior distribution)

Subscripts

calc calculated value using a theory
Ptrue possible true estimate reflecting epistemic uncertainty of estimation process
test measured value from a test
true true value

structural element in order to reduce the combined uncertainty. The remaining uncertainty after tests depends on the numbers of coupons and elements. Coupon tests are relatively cheap compared to element tests, and therefore, we usually perform many more coupon tests (several dozens) than element tests (a handful). The objective of this paper is to model the effect of these tests on making a conservative element strength prediction with a 95% confidence level by quantifying the uncertainty in the prediction process and to analyze the tradeoff between the number of coupons and elements for reducing the conservativeness.

In the overview of future structure technology for military aircraft, Joseph et al. [6] noted that a progressive uncertainty reduction model, which is seen in building-block tests, can be a feasible solution today, since a complete replacement of traditional tests with computational models is not feasible yet. Lincoln et al. [7] pointed out that building-block tests play a key role in reducing errors in failure prediction of composite structures due to large uncertainty in computational models. They noted that the use of probabilistic methods can significantly lower the test cost by reducing the scope of the test program.

There are also several studies investigating the effect of tests on safety and reducing uncertainty in computational models. Jiao and Moan [8] investigated the effect of proof tests on structural safety using Bayesian inference. They showed that proof tests reduce uncertainty in the strength of a structure, and thus provide a substantial reduction in the probability of failure. An et al. [9] investigated the effect of structural element tests on reducing uncertainty in element strength using Bayesian inference. Acar et al. [10] modeled a simplified building-block process with safety factors and knockdown factors. Bayesian inference is used to model the effect of structural element tests. They show the effect of the number of tests on the design weight for the same probability of failure, and vice versa. Jiang and Mahadevan [11] studied the effect of tests in validating a computational model by obtaining an expected risk in terms of the decision cost. Urbina and Mahadevan [12] assessed the effects of system level tests for assessing reliability of complex systems. They built computational models of a system and predicted the performance of the system. Tests are then incorporated into the models to estimate the confidence in the performance of the systems. Park et al. [13] estimated uncertainty in computational models and developed a methodology to evaluate likelihood using both test data and a computational model. McFarland and Bichon [14] estimated probability of failure by incorporating test data for a bistable MEMS device.

In this paper, we assume that with an infinite number of coupons and elements, the epistemic uncertainty associated with samples and failure theory can be eliminated. With a finite number of tests, the epistemic uncertainty is compensated for by using a conservative mean value at the 95% confidence level, in the