Reliability Engineering and System Safety 123 (2014) 137–144

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

AK-SYS: An adaptation of the AK-MCS method for system reliability

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ARTICLE INFO

Article history:
Received 31 January 2013
Received in revised form
24 October 2013
Accepted 31 October 2013
Available online 22 November 2013

Keywords:
System reliability
Kriging
Monte-Carlo simulation
Sampling
Classification
AK-MCS

ABSTRACT

A lot of research work has been proposed over the last two decades to evaluate the probability of failure of a structure involving a very time-consuming mechanical model. Surrogate model approaches based on Kriging, such as the Efficient Global Reliability Analysis (EGRA) or the Active learning and Kriging-based Monte-Carlo Simulation (AK-MCS) methods, are very efficient and each has advantages of its own. EGRA is well suited to evaluating small probabilities, as the surrogate can be used to classify any population. AK-MCS is built in relation to a given population and requires no optimization program for the active learning procedure to be performed. It is therefore easier to implement and more likely to spend computational effort on areas with a significant probability content. When assessing system reliability, analytical approaches and first-order approximation are widely used in the literature. However, in the present paper we rather focus on sampling techniques and, considering the recent adaptation of the EGRA method for systems, a strategy is presented to adapt the AK-MCS method for system reliability. The AK-SYS method, “Active learning and Kriging-based SYStem reliability method”, is presented. Its high efficiency and accuracy are illustrated via various examples.

1. Introduction

The reliability analysis of a structure or a system involves the evaluation of the probability that this system will fail, according to one or multiple failure modes that are carefully identified with specific methods which are outside the framework of this paper. Each one of the p modes is defined by a model, associated with a performance function $g_j(\cdot)$, $j=1,\ldots,p$, which delimits a failure domain $g_j(\mathbf{X})\le 0$, where $\mathbf{X}$ is the vector of random parameters. The probability of failure is defined as follows:

$$P_f=\int_{D_f} f_{\mathbf{x}}(\mathbf{x})\,d\mathbf{x} \tag{1}$$

where D_f is the system failure domain and $f_{\mathbf{x}}(\cdot)$ is the probability density function of the vector of random variables $\mathbf{X}$. The integral can be rearranged using the indicator function of the failure domain 1_{D_f}:

$$P_f=\int_{\mathbb{R}^n} 1_{D_f}(\mathbf{x})f_{\mathbf{x}}(\mathbf{x})\,d\mathbf{x} \tag{2}$$

$$1_{D_f}(\mathbf{x})=\begin{cases}1 & \text{if } \mathbf{x}\in D_f\\ 0 & \text{otherwise}\end{cases} \tag{3}$$

In practical cases, no exact solution can be found for this integral, which leads to the use of either analytical approaches or sampling techniques.

Let us first consider component reliability problems where $p=1$. On the one hand analytical methods typically employ a first- or second-order linearization (FORM or SORM, see [1,2]), of the limit state $g(\mathbf{X})=0$ in the vicinity of the Most Probable failure Point (MPP), found through the resolution of an optimization problem. The computational cost of such methods is rather low, but the accuracy of the result can also be low if the performance function is strongly non-linear in the vicinity of the MPP, or if other areas away from the identified MPP carry a significant probability content. On the other hand, sampling techniques use large populations, which are classified according to their sign on the performance function g. In the case of Monte-Carlo simulations [1,2], these populations are generated using the probability distribution associated with each variable. The probability of failure is then estimated as the number of realizations corresponding to failure with respect to the total number of evaluations denoted by N_{MC}:

$$P_f\approx\hat{P}_f=\frac{1}{N_{MC}}\sum_{i=1}^{N_{MC}} 1_{D_f}(\mathbf{x}^{(i)}) \tag{4}$$

Importance sampling [1,2] may be used in order to reduce the size of the population required to reach a small coefficient of variation on the estimation of P_f. It consists of centering the sampled population around the MPP. Although they can be very accurate, the main inconvenience with sampling techniques is their need for a relatively large number of performance function

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http://dx.doi.org/10.1016/j.ress.2013.10.010

evaluations. When these evaluations are time-consuming, which is usual in most engineering applications, surrogate model-based approaches such as polynomial response surfaces, polynomial chaos expansion (PCE), Support Vector Machine (SVM), or Gaussian process modeling, also known as Kriging, are a solution to substantially reduce the numerical cost of the reliability assessment. To this purpose, Kriging-based surrogate model approaches such as the EGRA method [3] and the AK-MCS method [4] have recently been introduced. Both are very accurate, efficient and have advantages of their own. EGRA is well suited to evaluating small probabilities, as the surrogate can be used to classify any population. AK-MCS is built in relation with a given population and requires no optimization program (on the whole design space) for the active learning procedure to be performed. It is therefore easier to implement and more likely to spend computational effort on areas with a significant probability content.

For system reliability, where $p>1$, both analytical approaches and sampling techniques can be adapted. FORM requires the calculation of multiple MPPs (for each limit state) and the related reliability indices, which are used as parameters in the multi-normal cumulative density function Φ_n (CDF) [5]. Various methods are available to evaluate Φ_n, but they can be expensive in terms of computational effort. As in the component case ($p=1$), the FORM linearization approximation may generate large errors. It is also possible to bound the system probability of failure using bounding inequalities, which involve component probabilities [6]. However these bounds can be rather large in practical engineering cases. Crude Monte-Carlo Sampling can be applied to carry out the classification on each performance function, and the system probability of failure can be calculated directly. However it involves a large number of calls to these functions. In the context of time-demanding performance functions, it is essential to reduce the computational effort, if sampling techniques are to be used. An adaptation of Kriging-based surrogate models for system reliability is available for the EGRA method in [7] and proposed for AK-MCS in the present paper. The purpose of this adaptation is to take advantage of the benefits of AK-MCS for system reliability.

Section 2 provides a summary on existing techniques for component reliability using Kriging-based surrogate models and discusses their useful features. It describes the principles constituting the EGRA and AK-MCS methods and puts them in perspective. Section 3 presents classic system reliability strategies to deal with multiple limit states constituting the failure domain, as well as the EGRA method for systems. Section 4 proposes an adaptation of the AK-MCS method, named AK-SYS, making it applicable to calculating the probability of failure of a system composed of simple series or parallel arrangements of failure modes. An illustration example is also presented. Section 5 illustrates the high efficiency and accuracy of the AK-SYS method with literature examples. The different results are compared with the EGRA method.

2. Kriging-based surrogate models for component reliability analysis

Reliability evaluation using sampling techniques requires that large populations be evaluated on the performance function, which can be prohibitively expensive in terms of computational effort. In order to classify random realizations according to their sign on the performance function, one solution is to replace the latter with a surrogate that can be evaluated inexpensively. The need for an accurate and efficient surrogate model led the authors in [3] to investigate Kriging. An active learning algorithm was proposed, in which the initial Kriging model is successively updated with new information from the true performance function, until it acquires the desired accuracy.

A Kriging predictor [8] is the realization of a stochastic field. It is an exact interpolator on the points used to build it, and the variance can be evaluated at each point of the simulation. Consequently, in reliability analysis, where only the sign of the prediction matters, the active learning objective is to add information in areas with a high uncertainty on the sign of the performance function. These particular features of Kriging are of great interest in building efficient surrogate models. In contrast with other surrogates approaches such as Polynomial Chaos Expension (PCE), computational effort can be focused on the limit state. Kriging-based models will be introduced in the following sections, for the EGRA and AK-MCS methods.

2.1. The EGRA method

The Efficient Global Reliability Analysis (EGRA) method, introduced in [3], is based on the following algorithm. First, a Kriging model is build on an initial random design of experiments, carried out on the true performance function. Then a learning function, labeled the Expected Feasibility Function (EFF), is used to estimate how likely the realization of the Kriging predictor at a particular point is expected to be close to the limit state $g(\mathbf{X})=0$, given its predicted value and variance. Point $\mathbf{x}^*$ which maximizes the EFF is the most "critical" point, *i.e.* with a substantial risk of a wrong estimation on the sign of g. An optimization program is employed to seek point $\mathbf{x}^*$ within the entire design space. The latter will then be evaluated using the true performance function and used to update the Kriging model. When sufficient information has been added to the model, a stopping criterion will be reached, guaranteeing the desired accuracy and ending the learning process. The algorithm is summarized hereinafter.

1. Random generation of an initial Design of Experiments (DoE) of size n_{DoE}.
2. Computation of the Kriging model $\hat{g}$ based on the n_{DoE} design sites $(\mathbf{x}^{(i)}, g(\mathbf{x}^{(i)}))$ using the DACE toolbox for Kriging [9].
3. Identification, within the whole design space and through the resolution of an optimization problem, of the maximum value on the Expected Feasibility Function (EFF), which defines the next best point $\mathbf{x}^*$ to evaluate on g.
4. If the stopping criterion has not been reached, $\mathbf{x}^*$ is evaluated on g, the original DoE is updated with $(\mathbf{x}^*, g(\mathbf{x}^*))$ and the algorithm goes back to step 2 for a new computation of the Kriging model. Otherwise, the surrogate has acquired sufficient accuracy.

Using the constructed meta-model, any sampled population can be classified to obtain an estimation of the probability of failure, which is an advantage of the EGRA method. However, to construct this surrogate, each learning step involves the resolution of an optimization problem and may waste computational effort on areas with low densities of probability.

2.2. The AK-MCS method

To save on computing resources, Active learning and Kriging-based Monte-Carlo Simulation (AK-MCS) [4] uses a Kriging surrogate model that is built specifically in accordance with a given Monte-Carlo population. The objective of this construction is to gain sufficient accuracy to classify this population, and not necessarily to refine the fit between the true performance and its surrogate over the whole design space. The active learning process is based upon a learning function evaluated only on the given and finite population, making it easier to implement without any optimization program, but also more likely to focus on areas