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Three higher order analytical nodal methods for multigroup neutron diffusion equations

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ABSTRACT

This work presents three efficient higher order analytical nodal methods for the numerical solution of a two-dimensional multigroup neutron diffusion equation in Cartesian geometry based on the use of the successive polynomial-weighted transverse integrations technique to convert a one-group diffusion equation to a system of coupled one-dimensional ordinary differential equations. These equations are then solved analytically over each homogenized cell after adequate approximations of the resulting effective sources after transversal integrations. Coupling between the approximate transverse flux-moments is achieved by imposing uniqueness constraint on their moments values. Adjacent elements are coupled by enforcing continuity conditions on the flux and current moments at interfaces cells. The weighted cell-balance equations and current-continuity conditions are then used to derive the discrete equations. These methods are applied for solving numerically various 2D benchmark problems and theirs performances discussed. Numerical results demonstrates more efficiency for the third higher order analytical nodal method for which the alone unknowns considered are the transverse flux moments on the interfaces of the homogenized elements.

1. Introduction

For the solution of the multigroup neutron diffusion equations in Cartesian geometry, we propose three higher order analytical nodal methods. Theses methods are all based on the use of successive polynomial-weighted transverse integrations technic (Ougouag and Rajic, 1988; Guessous and Akhmouch, 2002) to convert the original one-group diffusion equation to a system of coupled one dimensional ordinary differential equations. Theses equations are solved analytically over each homogenized cell. Coupling between the approximate transverse flux-moments in the two directions x and y is achieved by imposing consistent constraint relations on theirs moments values. Adjacent elements are coupled by enforcing continuity conditions on the moments of the flux and the current at the interfaces of cells. The cell-balance equations and the current-continuity conditions are then used to derive the discrete equations.

For the Partial Current Analytical Nodal Method of index k, PCANM-k, the principal unknowns are the outgoing-partial currents moments of order up to k at the faces of each homogeneous cell. The equations are formulated as response-matrix equations and the global system is solved naturally by a red–black Gauss–Seidel iterative method: For k fixed, we write for each homogeneous nodal element (or cell), the local response-matrix equations allowing to calculate the outgoing partial currents moments at the interfaces of nodal elements in terms of the incoming partial current moments and source terms. As the incoming partial current moments are the outgoing partial current moments of the adjacent nodes, the discrete equations leads naturally to an inner-iterative procedure of Gauss–Seidel type (Guessous and Akhmouch, 2002).

For the Flux Moments Analytical Nodal Method of index k, FANM-k, the principal unknowns are the cell-centered flux moments of order $(i,j), (i,j = 0,1,\ldots,k)$ and the edge flux moments of order up to k at the cell interfaces. An iterative procedure is deduced naturally to solve the resulted linear system of equations by de-coupling the set of cell-balance equations and the current continuity systems: At each iteration, the cell-flux moment estimates are substituted in the uncoupled, tridiagonal sets of x and y-current equations which can be solved independently of each other, so that the surface flux-moments can be computed. The resulting surface values are then substituted into the cell-balance equations to update the cell-flux moments. The formulation of the FANM-k can be considered as an extension of the lowest order Nodal Integration Method (NIM) (Azmy and Kirk, 1991) to higher order allowing the same facilities for the parallelization of the iterative solution procedure.

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The need of more precise results, suggest that the index k is augmented and/or the mesh is refined. The number of unknowns becomes then very large and computing cost more expensive specially when extension to three dimensional problems is to be performed. To surmount this drawback, we propose the Reduced Fluxes Moments Analytical Nodal Methods of Higher Order, RFANM-k, witch are simply a more satisfactory formulation of FANM-k: The centered flux moments are eliminated from the final discrete equations and the resulting linear system is only for the interfaces unknowns. Consequently, the number of unknowns is considerably diminished and the number of unknowns grow only as $O(k)$ instead $O(k^2)$. The reduced system of linear equations is solved by an ADI-like iterative procedure. After convergence, the centered cell parameters can be retrieved directly from the weighted balance equations. The RFANM-k methods can be very advantageous, specially for parallel implementation an three dimensional calculations.

Theses three nodal methods developed in this work have the capability to compute the in-node flux shapes by local expansion and demonstrates that with judicious choose of unknowns and adequate formulation of the resulting discrete equations, the elevation of the index $k \geqslant 1$ is no more an obstacle for computation, and can be very benefit for the solution of the steady state multigroup neutron diffusion equation.

Numerical results obtained for the 2D-PWR benchmark problems demonstrates the high performance of the three higher order analytical nodal methods, PCANM-k, FANM-k and RFANM-k and discuss the choice between increasing the index k or refine the mesh. Note that it would be interesting to compare the nodal methods developed in this article with the high-order nodal methods such as those developed by Ougouag and Rajic (1988), Altiparmakov and Tomašević (1990).

The plan of the paper is as follows. Some notations and preliminary of the statement of the problem are given in section two. In section three we give the procedures of discretization and iterative solution for each of the three nodal methods. Finally, the section four discusses numerical results and section five draws the conclusion.

2. Preliminaries and notations

2.1. The multigroup neutron diffusion equation

The distribution of the neutron flux in a reactor can be obtained by solving the multigroup neutron diffusion equations,

$$-\nabla.(D_g(r)\,\nabla\phi_g(r)) + \Sigma_g^t(r)\,\phi_g(r) = Q_g(r) \quad (r\in\Omega,\, g=1,\ldots,G) \tag{1}$$

The source term Q_g contains the fission and scattering contributions:

$$Q_g(r) = \frac{1}{\lambda}\chi_g \sum_{g'=1}^{G} \nu\Sigma_{g'}^f(r)\phi_{g'}(r) + \sum_{g'\neq g}^{G} \Sigma_{gg'}^s(r)\phi_{g'}(r) \tag{2}$$

The reactor domain Ω in 2-D cartesian geometry is a juxtaposition of large rectangular homogenized regions with piecewise constant properties, corresponding to fuel assemblies. The notations in Eqs. (1) and (2) are standard:

G = Total number of neutron energy groups.
λ = Effective multiplication factor (or k_{eff}).
χ_g = Total fission spectrum fraction in group g.
$\nu\Sigma_g^f$ = Macroscopic fission cross-section times the average number of secondaries, in group g.
$\Sigma_{gg'}^s$ = Macroscopic transfer cross-section from group g' to group g.
Σ_g^t = Macroscopic removal cross-section in group g
D_g = Diffusion coefficient in group g.
ϕ_g = Neutron flux in group g.

These equations are to be solved subject to the following homogeneous conditions at the reactor boundary $\partial\Omega$:

$$\alpha(r)\phi_g(r) + \beta(r)\overrightarrow{\nabla\phi_g}(r).\overrightarrow{n} = 0 \quad (r\in\partial\Omega,\ g=1,\ldots,G) \tag{3}$$

where α and β are nonnegative real constants such that $\alpha+\beta>0$. By an adequate choice of the constants this condition covers Dirichlet, Neumann and extrapolation boundary conditions and $\overrightarrow{n}$ is the outgoing unitary normal vector on $\partial\Omega$. It is convenient to rewrite Eqs. (1) and (2) into the following eigenvalue problem:

$$K\Phi = \frac{1}{\lambda}F\Phi \tag{4}$$

where $\Phi = (\phi_1,\ldots,\phi_G)^T$,

$$K = \begin{bmatrix} -\nabla.D_1\nabla+\Sigma_1^t & -\Sigma_{12}^s & \cdots & \cdots & -\Sigma_{1G}^s \\ -\Sigma_{21}^s & -\nabla.D_2\nabla+\Sigma_2^t & \cdots & \cdots & -\Sigma_{2G}^s \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ -\Sigma_{G1}^s & -\Sigma_{G2}^s & & & -\nabla.D_G\nabla+\Sigma_G^t \end{bmatrix}, \tag{5}$$

and $F = \chi\, B^T$. Where $\chi = (\chi_1,\ldots,\chi_G)^T$ and $B = (\nu\Sigma_1^f,\ldots,\nu\Sigma_G^f)^T$.

The eigenvalue Problem (4) is solved by the inverse power method (Wachspress, 1996). Starting with a positive, but otherwise arbitrary estimate $\Phi^{(0)}$ for the group fluxes, we generate successive estimates $\Phi^{(n)}$ and $\lambda^{(n)}$, $n=1,2,\ldots$, according to the following scheme:

$$\begin{cases} K\Psi^{(n)} = F\Phi^{(n-1)} \\ \lambda^{(n)} = \frac{(1,B^T\Psi^{(n)})}{(1,B^T\Phi^{(n-1)})} \\ \Phi^{(n)} = \frac{1}{\lambda^{(n)}}\Psi^{(n)} \end{cases} \tag{6}$$

In the absence of up-scattering ($\Sigma_{gg'}^s = 0, \quad for \quad g'>g$), at each step of this power iteration procedure we then solve G uncoupled self-adjoint elliptic boundary value problems of the form:

$$-\nabla.D\ \nabla\phi(x,y) + \Sigma\ \phi(x,y) = Q(x,y) \quad (x,y)\in\Omega \tag{7}$$

where Q is a known function. The unknown ϕ is subject to the same boundary condition Eq. (3).

2.2. Notations

The domain Ω is partitioned into M homogeneous rectangular cells (or nodal elements) Ω_m $(m=1,\ldots,M)$ forming a rectangular lattice. Let $\Omega_m = [-a,a]\times[-b,b]$, and the Legendre polynomial series $p_i(x)$, $i\in\mathbb{N}$ defined on $[-a,a]$ by $p_i(x) = P_i(\frac{x}{a})$, $i\in\mathbb{N}$ where $P_i(x)$ is the Legendre polynomial of degree i defined on $[-1,1]$, We have then the following orthogonality relations:

$$\int_{-a}^{a} p_i(x)p_j(x)dx = \frac{2a}{2i+1}\delta_{ij}, \quad i,j\in\mathbb{N} \tag{8}$$

where δ_{ij} is the Kronecker symbol.

The equation Eq. (7) is multiplied successively by the Legendre polynomial $p_j(y)$ $(j=0,\ldots,k)$ and integrated in the y-direction one $[-b,b]$. We then obtain a set of $(k+1)$ one-dimensional second order differential equation for the transverse flux moments $\phi_y^j(x)$, $(j=0,\ldots,k)$ in the x-direction.