Annals of Nuclear Energy 85 (2015) 426–433

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Dynamic Monte Carlo calculation method by solving frequency domain transport equation using the complex-valued weight Monte Carlo method

Toshihiro Yamamoto [a,*], Hiroki Sakamoto [b]

[a] Research Reactor Institute, Kyoto University, 2 Asashiro Nishi, Kumatori-cho, Sennan-gun, Osaka 590-0494, Japan
[b] Transnuclear Tokyo, Ltd., 1-18-16, Shinbashi, Minato-ku, Tokyo 105-0004, Japan

ARTICLE INFO

Article history:
Received 18 December 2014
Received in revised form 26 April 2015
Accepted 28 April 2015
Available online 14 May 2015

Keywords:
Monte Carlo
Fourier transformation
Frequency domain
Delayed neutron
ADS

ABSTRACT

A Monte Carlo method that calculates the transient behavior of neutron flux in the frequency domain is proposed in this study. A time-dependent neutron transport equation, in which the neutron source term varies over time while other properties remain constant, is Fourier transformed to obtain the transport equation in the frequency domain. The complex-valued transport equation in the frequency domain is subsequently solved with the complex-valued weight Monte Carlo method for each frequency contained in the Fourier transformed source intensity. The effect of delayed neutrons can be easily included in this frequency domain transport equation. Using the inverse Fourier transformation of the neutron flux in the frequency domain, we can obtain the time variation of the flux. This method is applicable to transient analyses of a subcritical system with a time-varying neutron source intensity. Several numerical examples indicate that the newly developed frequency domain calculation method provides good results compared to the time-dependent calculation method in the time domain. The computation time in the frequency domain is significantly shorter than that in the time domain, particularly for a nearly critical system.

1. Introduction

The transient analysis of a nuclear reactor is an important issue from many aspects in the field of the nuclear engineering. A point kinetics analysis is the most common and simple method used for this transient analysis. A variety of space- and time-dependent methods have been widely developed and utilized in nuclear reactor analyses; however, these kinetics analyses are primarily based on the deterministic method. Many approximations of neutron energy and geometry are introduced in the deterministic method. The continuous energy Monte Carlo method produces rigorous kinetics analyses with few approximations. Notable previous works on kinetics analyses with the Monte Carlo method utilize the Monte Carlo method to calculate the static neutron flux distribution in the improved quasi-static approximation (Goluoglu et al., 1998). In this method, the time variation of the reactor power is calculated by the deterministic point-kinetics equation.

Recently, the "dynamic Monte Carlo method" that uses only Monte Carlo techniques has been developed for nuclear reactor kinetics calculations (Sjenitzer and Hoogenboom, 2011, 2013; Sjenitzer et al., 2015). In this "dynamic Monte Carlo method", some new techniques were developed, such as the "forced decay for precursors" and "the branchless method". This method can yield satisfactory results for transient analyses in a reactor core compared with the conventional deterministic method. However, this new method requires a large amount of computational resources; for example, 40 CPU cores spent 200 h to calculate a transient phenomenon that lasts ~10 s (Sjenitzer and Hoogenboom, 2013).

This paper focuses on the transient behavior in a subcritical core, such as an accelerator driven system (ADS). In this paper, it is assumed that a transient phenomenon is induced by the time change of a neutron source intensity and that any other parameters remain unchanged. The transient phenomena induced by a source's time change have been investigated in the field of transient radiative transfer. Optical diagnostics of absorbing and scattering media illuminated by short-pulsed lasers is one application of transient radiative transfer (Elaloufi et al., 2002). A short-pulsed laser transport in an absorbing and scattering medium can be accurately analyzed by solving a time-dependent radiative transfer equation (TRTE). A wide variety of computational methods are available to solve this equation; a common method

* Corresponding author. Tel.: +81 72 451 2414; fax: +81 72 451 2658.
E-mail address: toshihiro.yamamoto223@gmail.com (T. Yamamoto).

http://dx.doi.org/10.1016/j.anucene.2015.04.041
0306-4549/

is the discrete ordinates finite-volume method (DO-FV) (Francoeur et al., 2005; Francoeur and Rousse, 2007). It is well known that a direct solution of a TRTE with the DO-FV method provides non-physical results because the transmitted radiative fluxes emerge earlier than the minimal time required by the radiation to leave the medium. This glitch is caused by the interdependence between the spatial and time discretizations and by the numerical approximations embedded within the application of an interpolation scheme (Francoeur and Rousse, 2007). This interdependence can be avoided by solving the transient radiative transfer problem in the frequency domain because no time-derivative term exists in the frequency domain equation. The time-dependent radiative transfer equation is transformed into the frequency domain equation by applying a Fourier transform. The complex-valued frequency domain equation is solved for each frequency contained in the incident pulse. Time-dependent results can be obtained by applying an inverse Fourier transform on the results in the frequency domain. This technique can successfully avoid physically unrealistic results in transient radiative transfer problems.

One of the difficulties in applying Monte Carlo techniques to reactor kinetic calculations lies in how to treat delayed neutrons. The lifetime of delayed neutron precursors ranges from $\sim 10^{-2}$ to 10^{2} s. Conversely, the lifetime of prompt neutrons is no longer than $\sim 10^{-3}$ s. To represent delayed neutrons in reactor kinetics calculations, two different time scale phenomena must be explicitly treated in the Monte Carlo calculations. After a prompt neutron and its progeny rapidly vanish, we must wait until the decay of the delayed neutron precursors. Because of the inconsistency in the time scales of prompt and delayed neutrons, at least several minute phenomenon must be analyzed even when we are interested in phenomenon operating over much shorter time durations.

However, Fourier transformations of the time-dependent equations for neutron transport and delayed neutron precursor densities yield frequency domain equations, which can subsequently be combined into one equation that does not explicitly include the delayed neutron precursor densities. This technique has been utilized in the neutron noise analysis technique in the frequency domain (Behringer et al., 1977, 1979; Demazière, 2004, 2011; Pázsit and Demazière, 2010; Larsson et al., 2011; Larsson and Demazière, 2012a,b). The frequency domain equation has the same form as that of a fixed source problem except that complex-valued terms are added, including the neutron flux. Conventionally, the neutron noise equation in the frequency domain has been based on diffusion theory. Recently, a method to solve the complex-valued equation with the Monte Carlo method was proposed by the authors of the present paper (Yamamoto, 2012, 2013, 2014; Yamamoto and Sakamoto, 2014).

By converting a time domain neutron transport equation into the frequency domain, the discretization in the time dimension can be eliminated. Thus, the frequency domain calculation is free from accumulation of an error associated with the time progression. Furthermore, a frequency domain calculation need not be aware of the large difference of the time scales between prompt and delayed neutrons. These two features make the Monte Carlo method easier to solve a transient behavior of nuclear fission chain reaction with delayed neutrons.

This paper addresses Monte Carlo transient analyses in a subcritical core by solving the complex-valued frequency domain neutron transport equation with delayed neutrons. Time-dependent neutron behavior induced by a neutron source term's time change is obtained by the inverse Fourier transformation of the solution of the frequency domain neutron transport equation. In the section that follows, this new Monte Carlo method is compared with the conventional time-dependent analysis method based on computational efficiency and accuracy.

2. Time-dependent analysis method in the frequency domain

We start with the time-dependent neutron transport equation with delayed neutrons:

$$\begin{aligned}
&\frac{1}{\upsilon(E)}\frac{\partial}{\partial t}\phi(\mathbf{r},\mathbf{\Omega},E,t)\\
&= -\mathbf{\Omega}\cdot\nabla\phi(\mathbf{r},\mathbf{\Omega},E,t) - \Sigma_t(\mathbf{r},E)\phi(\mathbf{r},\mathbf{\Omega},E,t)\\
&+ \int_{4\pi} d\mathbf{\Omega}' \int dE'\Sigma_s(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega},E'\to E)\phi(\mathbf{r},\mathbf{\Omega}',E',t)\\
&+ (1-\beta)\frac{\chi_p(E)}{4\pi}\int_{4\pi} d\mathbf{\Omega}'\int dE'\,\nu\Sigma_f(\mathbf{r},E')\phi(\mathbf{r},\mathbf{\Omega}',E',t) + \frac{\chi_d(E)}{4\pi}\lambda C(\mathbf{r},t)\\
&+ S(\mathbf{r},\mathbf{\Omega},E,t),
\end{aligned} \tag{1}$$

where $\phi(\mathbf{r},\mathbf{\Omega},E,t)$ = the neutron flux at position $\mathbf{r}$ with energy E, direction $\mathbf{\Omega}$, and time t; υ = the neutron velocity; Σ_t = the macroscopic total cross section; Σ_s = the macroscopic scattering cross section; Σ_f = the macroscopic fission cross section; $\chi_p(E)$ = the prompt neutron spectrum; $\chi_d(E)$ = the delayed neutron spectrum; ν = the number of neutrons per fission, β = the delayed neutron fraction, C = the density of the delayed neutron precursors; λ = the decay constant of the delayed neutron precursors; and S = the neutron source intensity. For simplicity, we assume one group of delayed neutrons is present. The time-dependent equation for the delayed neutron precursor density is thus:

$$\frac{\partial}{\partial t}C(\mathbf{r},t) = \beta\int_{4\pi} d\mathbf{\Omega}'\int dE'\,\nu\Sigma_f(\mathbf{r},E')\phi(\mathbf{r},\mathbf{\Omega}',E',t) - \lambda C(\mathbf{r},t). \tag{2}$$

In Eqs. (1) and (2), the time-dependent variables are $\phi(\mathbf{r},\mathbf{\Omega},E,t)$, $C(\mathbf{r},t)$, and $S(\mathbf{r},\mathbf{\Omega},E,t)$; the other variables such as the cross sections are considered constant. It is assumed that the neutron flux change is induced by the time variation of the neutron source intensity $S(\mathbf{r},\mathbf{\Omega},E,t)$.

The time domain equations, Eqs. (1) and (2), are converted to frequency domain equations by Fourier transformation of Eqs. (1) and (2). By eliminating the terms of the delayed neutron precursors from the Fourier transforms of Eqs. (1) and (2), we obtain the transport equation for the neutron flux in the frequency domain:

$$\begin{aligned}
&\mathbf{\Omega}\cdot\nabla\tilde{\phi}(\mathbf{r},\mathbf{\Omega},E,\omega) + \Sigma_t(\mathbf{r},E)\tilde{\phi}(\mathbf{r},\mathbf{\Omega},E,\omega)\\
&= \int_{4\pi} d\mathbf{\Omega}'\int dE'\Sigma_s(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega},E'\to E)\tilde{\phi}(\mathbf{r},\mathbf{\Omega}',E',\omega)\\
&+ \frac{1}{4\pi}\left((1-\beta)\chi_p(E) + \frac{\beta\lambda\chi_d(E)}{i\omega+\lambda}\right)\int_{4\pi} d\mathbf{\Omega}'\int dE'\,\nu\Sigma_f(\mathbf{r},E')\tilde{\phi}(\mathbf{r},\mathbf{\Omega}',E',\omega)\\
&- \frac{i\omega}{\upsilon(E)}\tilde{\phi}(\mathbf{r},\mathbf{\Omega},E,\omega) + \tilde{S}(\mathbf{r},\mathbf{\Omega},E,\omega),
\end{aligned} \tag{3}$$

where ω = angular frequency, $i = \sqrt{-1}$, and:

$$\tilde{\phi}(\mathbf{r},\mathbf{\Omega},E,\omega) \equiv \int_{-\infty}^{+\infty}\phi(\mathbf{r},\mathbf{\Omega},E,t)\,e^{-i\omega t}dt, \tag{4}$$

$$\tilde{S}(\mathbf{r},\mathbf{\Omega},E,\omega) \equiv \int_{-\infty}^{+\infty}S(\mathbf{r},\mathbf{\Omega},E,t)\,e^{-i\omega t}dt, \tag{5}$$

Note that the tilde denotes a complex-valued quantity. Eq. (3) is a fixed source equation in the frequency domain. We obtain $\tilde{\phi}(\mathbf{r},\mathbf{\Omega},E,\omega)$ by solving Eq. (3) at each frequency ω. Once we know $\tilde{\phi}(\mathbf{r},\mathbf{\Omega},E,\omega)$ for each frequency, the time-dependent neutron flux, $\phi(\mathbf{r},\mathbf{\Omega},E,t)$, can be obtained by the inverse Fourier transformation of $\tilde{\phi}(\mathbf{r},\mathbf{\Omega},E,\omega)$ as: