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Technical note

# Neutron spectrum kinetics in the infinite homogeneous reactor

Aldo Dall'Osso *

*Areva NP, Tour Areva, 92084 Paris la Défense Cedex, France*

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ABSTRACT

An analysis of the behaviour of the neutron spectrum in transient condition, based on an extension of the inhour equation to the energy dimension, is presented. This approach enables to describe the spectrum evolution in time as a combination of eigenstates associated to the roots of the inhour equation. The fine description of the neutron distribution in energy during the transient is exploited to evaluate homogenization errors committed by few-energy-group models.



## 1. Introduction

The kinetics of the infinite homogeneous reactor has been extensively explored and analytical solutions have been provided for the mono energetic neutrons (Keepin, 1965; Lamarsh, 1966; Akcasu et al., 1971; Ott and Neuhold, 1985). Analytical solutions are useful to setup benchmarks (Picca et al., 2013; Ganapol, 2013) validating neutronic transient codes. Analytical solutions are also useful to understand the physics of the neutron kinetics. However the analytical approach has been neglected for the neutron spectrum. One of the rare analytical approaches to the solution of the kinetic problem in multi-group theory was proposed by Schwalm (1965) to estimate the neutron spectrum in a pulsed experiment. The present work aims to give an analytical solution to the neutron spectrum kinetics by extending the inhour equation to the energy dimension. The evolution of the spectrum in the time is seen as a combination of eigenstates of the inhour equation.

## 2. Analysis of the spectrum in transient conditions

### 2.1. The spectrum in transient conditions

We write the kinetic neutron balance equation in infinite medium condition with an external source of intensity $s$ and spectrum $X_s$ for the multigroup formulation ($N_G$ energy groups) as:

$$V^{-1}\cdot\frac{d\Phi}{dt}=(1-\beta_{tot})F\cdot\Phi X_p-A\cdot\Phi+sX_s+\sum_{i=1}^{N_d}\lambda_i C_i X_{d,i} \tag{1a}$$

and for the $N_d$ precursors of delayed neutrons ($i = 1,\ldots, N_d$):

$$\frac{dC_i}{dt}=\beta_i F\cdot\Phi-\lambda_i C_i, \tag{1b}$$

with

$$\beta_{tot}=\sum_{i=1}^{N_d}\beta_i. \tag{2}$$

The following initial conditions are associated:

$$\Phi(0)=\Phi_0, \tag{3a}$$

$$C_i(0)=C_{0,i}. \tag{3b}$$

Operator $A$ is the absorption and scattering matrix, $X_p$ and $X_{d,n}$ ($n = 1,\ldots,N_d$) are the emission spectrum vectors of the prompt and delayed neutrons respectively, $F$ is the fission neutron emission vector and $V^{-1}$ is the diagonal matrix having the inverse of the neutron velocity $v_i$ ($i = 1,\ldots,N_G$) as elements:

$$\begin{aligned}
A&=\text{matrix}\left[-\Sigma_{i,j}+\delta_{i,j}\left(\Sigma_{a,i}+\sum_{k=1}^{N_G}\Sigma_{k,i}\right)\right],\\
X_p&=\text{vector}\left[\chi_{p,i}\right],\\
X_{d,n}&=\text{vector}\left[\chi_{d,n,i}\right],\\
F&=\text{vector}\left[\nu\Sigma_{f,i}\right],\\
V^{-1}&=\text{matrix}\left[\delta_{i,j}/v_i\right],\\
\Phi&=\text{vector}[\varphi_i],\qquad (i,j=1,\ldots,N_G).
\end{aligned} \tag{4}$$

* Tel.: +33 1 34 96 7649; fax: +33 1 34 96 7669.
*E-mail address:* aldo.dallosso@areva.com

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The convention to express with $\Sigma_{i,j}$ the neutrons coming from group $j$ to group $i$ has been adopted. Symbol $\delta_{i,j}$ is the Kronecker delta.

Assuming the medium properties do not change after $t = 0$, a possible solution can be expressed as a combination of exponentials:

$$\Phi(t) = \sum_{j=0}^{N_d} a_j \varphi_j e^{\omega_j t}, \tag{5a}$$

$$C_i(t) = \sum_{j=0}^{N_d} c_{i,j} e^{\omega_j t}, \tag{5b}$$

which implies that the initial conditions can be expressed as:

$$\sum_{j=0}^{N_d} a_j \varphi_j = \Phi_0, \tag{6a}$$

$$\sum_{j=0}^{N_d} c_{i,j} = C_{0,i}, \tag{6b}$$

provided that the initial spectrum can be decomposed on the $\varphi_j$ basis functions. In Section 2.4 the conditions in which Eq. (5) is complete will be discussed.

### 2.2. The inhour equation for the neutron spectrum

Substituting Eqs. (5a) and (5b) into Eq. (1b) we obtain:

$$\sum_{j=0}^{N_d} e^{t\omega_j}\omega_j c_{i,j} = \beta_i \sum_{j=0}^{N_d} e^{t\omega_j} F\cdot\varphi_j a_j - \lambda_i \sum_{j=0}^{N_d} e^{t\omega_j} c_{i,j}. \tag{7}$$

Since this equation must hold for any time $t$, we have one equation for each exponential

$$\omega_j c_{i,j} = F\cdot\varphi_j a_j \beta_i - \lambda_i c_{i,j}, \tag{8}$$

which solved for $c_{i,j}$ gives

$$c_{i,j} = \frac{F\cdot\varphi_j a_j \beta_i}{\lambda_i + \omega_j}. \tag{9}$$

Substituting Eqs. (5a), (5b) and (9) into Eq. (1a) we obtain, in absence of external source:

$$V^{-1}\cdot\sum_{j=0}^{N_d} e^{t\omega_j} a_j\varphi_j\omega_j = -\sum_{j=0}^{N_d} e^{t\omega_j} A\cdot\varphi_j a_j + X_p(1-\beta_{tot})\sum_{j=0}^{N_d} e^{t\omega_j} F\cdot\varphi_j a_j + \sum_{i=1}^{N_d}\lambda_i X_{d,i}\sum_{j=0}^{N_d}\frac{e^{t\omega_j}F\cdot\varphi_j a_j\beta_i}{\lambda_i+\omega_j}. \tag{10}$$

Again, the requirement to satisfy this equation for any time $t$ leads to

$$\left(A + V^{-1}\omega_j\right)\cdot\varphi_j = F\cdot\varphi_j\left(X_p(1-\beta_{tot}) + \sum_{i=1}^{N_d}\frac{\beta_i\lambda_i X_{d,i}}{\lambda_i+\omega_j}\right). \tag{11}$$

This is the equation which provides the eigenstates of the kinetic problem. Eq. (11) can be simplified after some manipulation. Let us define the adjoint spectrum as solution of:

$$A^*\cdot\Phi^* = \frac{X_{av}\cdot\Phi^*}{k_\infty}F, \tag{12}$$

where $X_{av}$ is the average emission spectrum:

$$X_{av} = X_p(1-\beta_{tot}) + \sum_{i=1}^{N_d}\beta_i X_{d,i}. \tag{13}$$

Multiplying Eq. (11) by $\Phi^*$ and Eq. (12) by $\varphi_j$, subtracting and taking into account that, in virtue of the definition of adjoint operator, the following relationship holds:

$$\varphi_j\cdot A^*\cdot\Phi^* = \Phi^*\cdot A\cdot\varphi_j, \tag{14}$$

we obtain:

$$\Phi^*\cdot\sum_{i=1}^{N_d}\frac{\beta_i\lambda_i X_{d,i}}{\lambda_i+\omega_j} = \frac{\Phi^*\cdot X_{av}}{k_\infty} - \Phi^*\cdot X_p(1-\beta_{tot}) + \frac{\Phi^*\cdot V^{-1}\cdot\varphi_j\omega_j}{F\cdot\varphi_j}. \tag{15}$$

The above expression can be further transformed using the definitions of:

- reactivity:

$$k_\infty = \frac{1}{1-\varrho}, \tag{16}$$

- neutron generation time associated to the $j$-th component:

$$\Lambda_j = \frac{\Phi^*\cdot V^{-1}\cdot\varphi_j}{F\cdot\varphi_j\Phi^*\cdot X_{av}}, \tag{17}$$

- effectiveness factor for the $i$-th delayed neutron group (see Section 3.1):

$$\gamma_i = \frac{\Phi^*\cdot X_{d,i}}{\Phi^*\cdot X_{av}}. \tag{18}$$

Finally, solving with respect to $\varrho$ we obtain the inhour equation:

$$\varrho = \omega_j\left(\Lambda_j + \sum_{i=1}^{N_d}\frac{\beta_i\gamma_i}{\lambda_i+\omega_j}\right). \tag{19}$$

This equation differs from the classical one for mono-energetic neutrons (Nordheim, 1946) for having the effectiveness factor $\gamma_i$ as weighting factor for the contribution of the delayed neutrons. Within a multi-energy context, the inhour equation can be considered as constituted by Eq. (11), which provides the eigenfunctions and Eq. (19), whose roots are the eigenvalues.

### 2.3. Determination of the constants

To determine the constants of the flux solution, we follow the approach proposed by Lamarsh (1966). We suppose that the reactor is initially critical, therefore Eq. (1b) gives:

$$C_{0,i} = \frac{F\cdot\Phi_0\beta_i}{\lambda_i}. \tag{20}$$

However from Eqs. (9) and (6b) we have:

$$C_{0,i} = \sum_{j=0}^{N_d}\frac{F\cdot\varphi_j a_j\beta_i}{\lambda_i+\omega_j}. \tag{21}$$

Comparing Eqs. (20) and (21) we can write the following set of equations ($i = 1,\ldots,N_d$):

$$\frac{F\cdot\Phi_0}{\lambda_i} = \sum_{j=0}^{N_d}\frac{F\cdot\varphi_j a_j}{\lambda_i+\omega_j}. \tag{22}$$

Another equation is needed to close the system. It can be obtained projecting Eq. (6a) on an arbitrary function, which we chose as the adjoint flux:

$$\sum_{j=0}^{N_d}\Phi^*\cdot\varphi_j a_j = \Phi^*\cdot\Phi_0. \tag{23}$$

The solution of the system composed by Eqs. (22) and (23) provides the coefficients of the expansion in Eq. (5a). Once the $a_j$ coefficients have been computed, the components $c_{i,j}$ of the precursors densities are obtained applying Eq. (9).