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Minimum critical mass in coupled core systems

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ABSTRACT

Minimum critical mass configurations of well moderated coupled core systems are studied for the moderators heavy water, beryllium and graphite. Characteristic for these systems are the delta functions in the fuel distributions at the core boundaries facing each other and the occurrence of minima in the core size, both depending on the moderator properties and the inter core distance. For an ideal moderator like pure heavy water, the minimum critical mass configuration of the fuel consists of two delta functions for a range of inter core distances.

1. Introduction

The design of a critical system with a minimum amount of fissile material is a classical problem in reactor physics. The seminal paper in this field is authored by Goertzel (1956). He presented an elegant proof that in the framework of diffusion theory and in a well moderated system with the same moderator in core and reflector, the fuel importance is proportional to the thermal flux; this implies that a minimum critical mass (from now on abbreviated as MCM) requires a fuel distribution that leads to a flat thermal flux in the core, which theorem is later named after him. Since then many researchers contributed to the MCM problem; an extensive list of references is given by Williams (2004). An important finding in the last reference concerns a peculiar consequence of the Goertzel theorem. If one requires that the thermal flux is flat in the core region, this leads for every fuel/moderator combination to a well-defined core size with MCM. However, if one requires a core size smaller than this, the case of so-called restricted MCM, this requires an unphysical fuel distribution with a delta function at the core–reflector interface. With a restricted core size, the thermal neutron current from reflector to the core is higher than vice versa which would lead to a flux gradient near the interface; the surplus thermal current from reflector to core must in that case be absorbed by adding a delta distribution to the fuel at the interface. This means an infinitely thin sheet of fuel with infinite density which is of course not physical. Williams (2004) showed that this is a consequence of diffusion theory; by presenting a transport theory analysis he showed that the delta distribution is replaced by a steeply increasing (but still diverging) fuel density near the interface.

While the Goertzel theorem refers to systems with a continuous fuel distribution, a paper by the present author (Van Dam, 2013) presents an extension for heterogeneous systems with thin fuel plates, that allow an analysis by treating the fuel sheets with the so-called Feinberg–Galanin–Horning method of heterogeneous reactors, most recently explained in Williams (2000).

In the heterogeneous case MCM is realized by positioning the fuel sheets in such a way that, in addition to thermal flux equality at the sheets, the so-called thermal current balance condition at each sheet is satisfied, which means equal currents at both sides of the sheet. By these conditions both the individual positions and thermal absorption strengths of the fuel sheets are fixed.

Williams presented in (Williams, 2003) another interesting "anomalous" phenomenon: in MCM systems with finite reflectors and a continuous fuel distribution the critical core size can take the same value for two different reflector thicknesses. A physical explanation of this fact was presented in Van Dam (accepted for publication).

The present paper focusses on MCM systems with coupled cores. This means that we consider two well moderated and reflected cores, separated by a layer of the same moderator with such a thickness that neutrons from one core can interact in the other core. In this layer the low thermal neutron absorption in a good moderator leads to an increase in thermal flux; for this reason the layer is in literature often indicated as "flux trap". This is a

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special case of restricted MCM because the thickness of the flux trap is taken as fixed while for absolute MCM the flux trap should be absent. From physical considerations we expect that the high thermal neutron current from the flux trap to the cores necessitates a "delta layer" of fuel at the core–trap interfaces and that the critical core thickness as a function of trap thickness will show a phenomenon comparable to the anomaly discovered by Williams in 2003.

2. Theory

In the framework of the Goertzel theorem we adopt a 2-group diffusion model for a well thermalized system without absorption of non-thermal neutrons, well applicable for a system with pure U-235 as a fuel. In addition we consider a one-dimensional slab geometry.

The two-energy group equations for a critical system are:

$$D_1 \frac{d^2}{dx^2}\phi_1 - \Sigma_{12}\phi_1 + \eta\Sigma_{af}(x)\phi_2 = 0 \tag{1a}$$

$$D_2 \frac{d^2}{dx^2}\phi_2 + \Sigma_{12}\phi_1 - (\Sigma_{af}(x) + \Sigma_m)\phi_2 = 0 \tag{1b}$$

where the subscripts 1 and 2 refer to the fast and thermal group, respectively. The diffusion coefficients D refer to the pure moderator; this is valid for very diluted U-235 in a high quality moderator so all macroscopic moderator parameters are space independent. Σ_{12} is the moderation cross section, Σ_m is the absorption cross section of the moderator, Σ_{af} is the macroscopic thermal absorption (fission + capture) cross section of the fuel (the only space dependent nuclear parameter in this case) and η the average number of fission neutrons emitted per absorbed thermal neutron in the fuel. The independent space variable x is omitted at the fluxes for simplicity of notation.

We put the origin of the x-coordinate at the symmetry axis of the system and describe half of the system as follows:

From $x = 0$ to $x = t$: the internal moderator or "trap".

From $x = t$ to $x = t + c$: the core region.

From $x = t + c$ to $x = T$: the reflector, T being the extrapolated half system thickness.

The general solution for the fast flux is:

$$0 \leqslant x \leqslant t: \qquad \varnothing_1 = A\cosh(\kappa_1 x) \tag{2a}$$

$$t \leqslant x \leqslant t + c: \qquad \varnothing_1 = B\cos(\lambda x) + C\sin(\lambda x) + \frac{\eta\Sigma_m}{(\eta - 1)\Sigma_{12}} \tag{2b}$$

$$t + c \leqslant x \leqslant T: \qquad \varnothing_1 = E\sinh[\kappa_1(T - x)] \tag{2c}$$

where $\kappa_1 = \sqrt{D_1/\Sigma_{12}}$.

Eq. (2b) is obtained as follows: because of the linearity of the system, the absolute fluxes are arbitrary, so we assume a unit thermal flux in the core. As a consequence, Eq. (1b) in the core gives:

$$\Sigma_{af}(x) = \Sigma_{12}\phi_1(x) - \Sigma_m \tag{2d}$$

Inserting (2d) into (1a) gives the equation for the fast flux in the core:

$$D_1 \frac{d^2}{dx^2}\phi_1 + (\eta - 1)\Sigma_{12}\phi_1 - \eta\Sigma_m = 0. \tag{2e}$$

The homogeneous part is satisfied by sine and cosine solutions, where $\lambda = \kappa_1\sqrt{\eta - 1}$, a particular solution of the inhomogeneous equation is the last term in Eq. (2b).

In Eqs. (2a–2c) we have already taken into account requirements of symmetry and vanishing flux at the extrapolated outer boundary.

Next we have to apply boundary conditions to the two inner boundaries. The inner boundary between flux trap and core can contain a delta layer of fuel with strength Λ, the position of the outer core boundary results from the criticality condition for the system.

Continuity of fast flux at $x = t$ gives:

$$A\cosh(\kappa_1 t) = B\cos(\lambda t) + C\sin(\lambda t) + \frac{\eta\Sigma_m}{(\eta - 1)\Sigma_{12}} \tag{3a}$$

The fast current densities at $x = t$, taking into account the fast neutron production in the Λ layer by the unit thermal flux, should obey:

$$\kappa_1 D_1 A\sinh(\kappa_1 t) + \lambda D_1 B\sin(\lambda t) - \lambda D_1 C\cos(\lambda t) = \eta\Lambda \tag{3b}$$

Continuity of flux and current at $x = t + c$ gives:

$$B\cos[\lambda(t + c)] + C\sin[\lambda(t + c)] + \frac{\eta\Sigma_m}{(\eta - 1)\Sigma_{12}} = E\sinh[\kappa_1(T - t - c)] \tag{3c}$$

$$-\lambda B\sin[\lambda(t + c)] + \lambda C\cos[\lambda(t + c)] = -\kappa_1 E\cosh[\kappa_1(T - t - c)] \tag{3d}$$

For the thermal flux the general solutions are:

$$0 \leqslant x \leqslant t: \qquad \varnothing_2 = A\frac{\Sigma_{12}}{\Sigma_m - \kappa_1^2 D_2}\cosh(\kappa_1 x) + F\cosh(\kappa_2 x) \tag{4a}$$

$$t \leqslant x \leqslant t + c: \qquad \varnothing_2 = 1 \tag{4b}$$

$$t + c \leqslant x \leqslant T: \quad \varnothing_2 = E\frac{\Sigma_{12}}{\Sigma_m - \kappa_1^2 D_2}\sinh[\kappa_1(T - x)] + G\sinh[\kappa_2(T - x)] \tag{4c}$$

where $\kappa_2 = \sqrt{\Sigma_m/D_2}$.

Eq. (4b) reflects the aforementioned choice of a unit thermal flux in the core.

The boundary conditions are two for the flux continuities at the internal interfaces and two current balances, taking into account the Λ absorber layer for the thermal neutrons:

$$\varnothing_2(t) = A\frac{\Sigma_{12}}{\Sigma_m - \kappa_1^2 D_2}\cosh(\kappa_1 t) + F\cosh(\kappa_2 t) = 1 \tag{5a}$$

$$\varnothing_2(t + c) = E\frac{\Sigma_{12}}{\Sigma_m - \kappa_1^2 D_2}\sinh[\kappa_1(T - t - c)] + G\sinh[\kappa_2(T - t - c)] = 1 \tag{5b}$$

$$\varnothing_2'(t_-) = A\frac{\kappa_1\Sigma_{12}}{\Sigma_m - \kappa_1^2 D_2}\sinh(\kappa_1 t) + F\kappa_2\sinh(\kappa_2 t) = -\frac{\Lambda}{D_2} \tag{5c}$$

$$\varnothing_2'(t + c)_+ = -E\frac{\kappa_1\Sigma_{12}}{\Sigma_m - \kappa_1^2 D_2}\cosh[\kappa_1(T - t - c)] - G\kappa_2\cosh[\kappa_2(T - t - c)] = 0 \tag{5d}$$

where the indices "–" and "+" refer to immediately left and right from an interface, respectively.

Eq. (5d) expresses a continuity of thermal gradient (equal to zero) at the right core boundary. This is the condition for absolute MCM and implicitly determines the core size; in case we fix the core size, a delta function (positive or negative, the latter being physical) in the fuel distribution is needed at the outer core boundary.