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Multi-group SP_3 approximation for simulation of a three-dimensional PWR rod ejection accident

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ABSTRACT

Previous researchers have shown that the simplified P_3 (SP_3) approximation is capable of providing sufficiently high accuracy for both static and transient simulations for reactor core analysis with considerably less computational expense than higher order transport methods such as the discrete ordinate or the full spherical harmonics methods. The objective of this paper is to provide a consistent comparison of two-group (2G) and multi-group (MG) diffusion and SP_3 transport for rod ejection accident (REA) in a practical light water reactor (LWR) problem. The analysis is performed on two numerical benchmarks, a 3×3 assembly mini-core and a full pressurized water reactor (PWR) core. The calculations were performed using pin homogenized and assembly homogenized cross sections for a series of benchmarks of increasing difficulty, in two-dimensional (2D) and three-dimensional (3D), 2G and MG, diffusion and transport, as well as with and without feedback. All results show consistency with the reference results obtained from higher-order methods. It is demonstrated that the analyzed problems show small group-homogenization effects, but relatively significant transport effects which are satisfactorily addressed by the SP_3 transport method. The sensitivity tests also show that, for the REA simulation, the MG is more conservative than 2G, P_1 is more conservative than SP_3 for a 1/3 MOX loaded full-core problem.

1. Introduction

For several decades the diffusion approximation has successfully been applied for the analysis of the current generation of LWRs. For spatial discretization, advanced nodal methods such as the nodal expansion method (NEM) (Finnemann et al., 1977), the analytic nodal method (ANM) (Smith, 1979), the nodal integration method (NIM) (Fisher and Finnemann, 1981) and the analytic function expansion method (AFEN) (Noh and Cho, 1993) have been used successfully to design and analyze several generations of reactors. These methods have been able to predict fuel pin-powers within a few percent of measured data using assembly size computational mesh with some type of “pin-power reconstruction” technique.

However, there has been a concern that the methods which were developed and benchmarked primarily for Uranium fueled LWRs do not perform equally well when applied to mixed oxide (MOX) fuelled cores or other cores with a very heterogeneous fuel loading. Several researchers have identified the specific approximations that contribute to the errors observed in the nodal methods. Systematic analysis has isolated deficiencies in four basic categories: a spatial discretization effect, a spatial homogenization effect, a group collapsing effect, and a transport effect (Downar et al., 2000, 2002; Lee et al., 2002).

Because of these issues, the neutronics methods in the U.S. NRC neutron kinetics code PARCS (Purdue Advance Reactor Core Simulator) (Downar et al., 2002) were improved for the transient analysis of LWR cores with MOX fuel. There was concern that the conventional 2G nodal methods could not properly treat partially fueled MOX cores which have large flux gradients at the interface of MOX and uranium-oxide (UOX) fuel assemblies. Downar et al. assessed the impact of MOX loading on the accuracy of the conventional 2G nodal approaches in PARCS based on the various approximations used to solve the Boltzmann transport equation: assembly homogenization, spatial discretization, group collapsing, transport effect and spatial dehomogenization (Downar et al., 2000). Subsequent research efforts addressed these issues and this paper summarizes the improvements in the PARCS methodology for MOX steady-state and transient core analysis.

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2. PARCS methodology for MOX analysis

A MG SP_3 approximation was added to PARCS for the analysis of MOX fuelled cores. The SP_3 method was implemented with both fine mesh finite difference and nodal spatial discretization options (García-Herranz et al., 1999, 2003). In the following section the time-dependent SP_3 equations are presented as implemented in PARCS and specific details for each kernel will follow.

2.1. Time-dependent SP_3 equations

Time-dependent SP_3 equations can be written as Eq. (1) (Brantley and Larsen, 2000; Shin and Miller, 1998)

$$\frac{1}{\upsilon_g}\frac{\partial\phi_{0g}}{\partial t}+\nabla\cdot\phi_{1g}+\Sigma_{rg}\phi_{0g}=s_{0tg}, \tag{1a}$$

$$\frac{1}{\upsilon_g}\frac{\partial\phi_{1g}}{\partial t}+\frac{2}{3}\nabla\phi_{2g}+\frac{1}{3}\nabla\phi_{0g}+\Sigma_{trg}\phi_{1g}=0, \tag{1b}$$

$$\frac{1}{\upsilon_g}\frac{\partial\phi_{2g}}{\partial t}+\frac{3}{5}\nabla\cdot\phi_{3g}+\frac{2}{5}\nabla\cdot\phi_{1g}+\Sigma_{tg}\phi_{2g}=0, \tag{1c}$$

$$\frac{1}{\upsilon_g}\frac{\partial\phi_{3g}}{\partial t}+\frac{3}{7}\nabla\phi_{2g}+\Sigma_{tg}\phi_{3g}=0, \tag{1d}$$

$$\frac{dC_k}{dt}=-\lambda_k C_k+\frac{\beta_k}{k_{eff}}\sum_{g'}\nu\Sigma_{fg'}\phi_{0g'} \tag{1e}$$

where

φ_{lg} is the l-th moment of neutron angular flux of group g,
υ_g is the neutron velocity of group g,
$\Sigma_{rg}, \Sigma_{trg}$ and Σ_{tg} are the removal, transport, and total cross sections of group g, respectively,
$s_{0tg}=\sum_{g'\neq g}\Sigma_{sg'g}\phi_{0g'}+\frac{\chi_g}{k_{eff}}(1-\beta)\sum_{g'}\nu\Sigma_{fg'}\phi_{0g'}+\chi_{dg}\sum_k\lambda_k C_k$ is the neutron source term,
$\Sigma_{sg'g}$ is the scattering source from group g' to g,
χ_g and χ_{dg} are the fission spectrum and delayed neutron spectrum,
k_{eff} is the multiplication factor,
β and β_k are the total and group k delayed neutron fractions,
ν is the number of neutrons per fission,
Σ_{fg} is the fission cross section of group g,
λ_k is the decay constant of the precursor delayed group k,
C_k is the precursor concentration of delayed group k,
$g(=1, 2, ..., \text{G})$ is the neutron energy group index,
and $k(=1, 2, ..., \text{K})$ is the delayed neutron precursor group index.

Substituting odd-moment flux into even-order equation and applying the conventional theta time-discretization method to Eq. (1) yields (Lee, 2001a)

$$\begin{bmatrix}-D_1^*\nabla^2+\Sigma_r^* & -2D_1^*\nabla^2\\ -\frac{2}{5}D_1^*\nabla^2 & -\left(\frac{3}{5}D_3^*+\frac{4}{5}D_1^*\right)\nabla^2+\Sigma_t^*\end{bmatrix}\begin{bmatrix}\phi_0^{n+1}\\ \phi_2^{n+1}\end{bmatrix}=\begin{bmatrix}p_0^n-3D_1^*\nabla\cdot p_1^n+s_{0t}^{n+1}\\ p_2^n-\frac{6}{5}D_1^*\nabla\cdot p_1^n-\frac{7}{5}D_3^*\nabla\cdot p_3^n\end{bmatrix} \tag{2}$$

where

$$D_1^*\equiv\frac{1}{3\Sigma_{tr}^*},$$

$$D_3^*\equiv\frac{3}{7\Sigma_t^*},$$

$$\Sigma_\alpha^*=\Sigma_\alpha+\frac{1}{\theta\upsilon\Delta t},$$

$$p_{ig}^n=q_{ig}^n+\bar{\Theta}r_{ig}^n,$$

$$q_{ig}^n=\frac{\phi_{ig}^n}{\theta\upsilon\Delta t},$$

$$\bar{\Theta}=\frac{1}{\theta}-1,$$

$$r_{0g}^n=\left(s_{0tg}-\nabla\cdot\phi_{1g}-\Sigma_{rg}\phi_{0g}\right)^n,$$

$$r_{1g}^n=\left(-\frac{2}{3}\nabla\phi_{2g}-\frac{1}{3}\nabla\phi_{0g}-\Sigma_{trg}\phi_{1g}\right)^n,$$

$$r_{2g}^n=\left(-\frac{3}{5}\nabla\cdot\phi_{3g}-\frac{2}{5}\nabla\cdot\phi_{1g}-\Sigma_{tg}\phi_{2g}\right)^n,$$

$$r_{3g}^n=\left(-\frac{3}{7}\nabla\phi_{2g}-\Sigma_{tg}\phi_{3g}\right)^n,$$

θ is the Crank–Nicolson time step parameter,and$n$ is the time step index.

For brevity, the delay neutron precursor equations for the SP_3 flux equations are not shown in Eq. (2) since they are identical to the original PARCS approach which uses analytic integration of precursor equations with the approximation of 2nd order polynomial fission source in time (Brantley and Larsen, 2000). Due to the p_{1g}^n and p_{3g}^n terms on the right hand side, Eq. (2) cannot be solved without additional equations for those two terms. However, if we accept the following approximations for application of the SP_3 equations to LWR analysis, which means that the time variation of odd flux moments much smaller than the spatial flux variations of even flux moments:

$$\frac{3}{\upsilon_g}\frac{\partial\phi_{1g}}{\partial t}<<\left(2\nabla\phi_{2g}+\nabla\phi_{0g}\right) \tag{3a}$$

$$\frac{7}{3}\frac{1}{\upsilon_g}\frac{\partial\phi_{3g}}{\partial t}<<\nabla\phi_{2g} \tag{3b}$$

then the time derivatives of 1st and 3rd moments can be ignored:

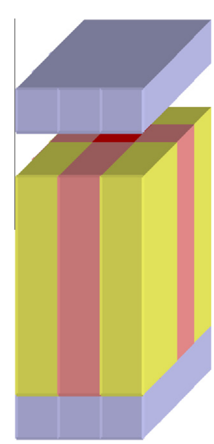

Fig. 1. 3 × 3 fuel assembly mini-core.