Annals of Nuclear Energy 77 (2015) 408–414

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Technical note

The influence of the neutron source spectrum on the infinite homogeneous reactor in subcritical condition

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ARTICLE INFO

Article history:
Received 5 November 2014
Accepted 7 December 2014

Keywords:
Infinite medium
Infinite reactor
Multiplication factor
k_∞
Subcritical condition

ABSTRACT

An analysis of the influence of the neutron source spectrum on some key parameters of a reactor system in subcritical condition is presented. The analysis is conducted on the infinite homogeneous reactor, a kind of approach useful to have a quick understanding of nuclear reactor phenomena. The main differences between the spectrum in critical and subcritical condition are highlighted. Among the results of the analysis we present the impact on the multiplication factor of neglecting the delayed neutron spectrum. The homogenization error on few-group cross sections due to using critical spectra in the standard homogenization procedures is also discussed.

1. Introduction

Subcritical reactor analysis is relevant not only for accelerator-driven systems (ADS) but also for critical reactors. Actually, safety analyses demand to know the behavior of the reactor during outage, when the reactor is maintained subcritical and the neutron spectrum is influenced by the source neutron emission spectrum. Research in the domain of subcritical states gives priority to the analysis of the spatial distribution of the neutrons, with respect to the distribution in the energy domain. However, reducing the neutron transport to the only energy dimension enables decoupling the neutron slowing down from complex spatial-directional interactions. Investigations on the response to a source in terms of the sole energy dimension permit to understand the effect of the energy distribution of the source on the neutron spectrum and subcritical multiplication factor.

2. Analysis of the subcritical spectrum

2.1. The dependence of the spectrum on the source

We write the inhomogeneous neutron balance equation in infinite medium condition with a source of magnitude f_s for the multigroup formulation (N_G energy groups) as:

$$A \cdot \Phi = F \cdot \Phi X_c + f_s X_s \tag{1}$$

where A is the absorption and scattering matrix, X_n is the neutron emission spectrum vector ($n = c$ for fission source and $n = s$ for external source) and F is the fission neutron production vector:

$$\begin{aligned} A &= \text{matrix}\left[-\Sigma_{i,j} + \delta_{i,j}\left(\Sigma_{a,i} + \sum_{k=1}^{N_G}\Sigma_{k,i}\right)\right], \\ X_n &= \text{vector}\left[\chi_{n,i}\right], \\ F &= \text{vector}\left[\nu\Sigma_{f,i}\right], \\ \Phi &= \text{vector}[\varphi_i], \quad (i,j = 1,\ldots,N_G). \end{aligned} \tag{2}$$

We have adopted the convention to express with $\Sigma_{i,j}$ the neutrons coming from group j to group i. Symbol $\delta_{i,j}$ is the Kronecker symbol.

If we define with f_c the total neutron production rate from fission (i.e. the internal source):

$$f_c = F \cdot \Phi, \tag{3}$$

Eq. (1) can be transformed into the following one:

$$A \cdot \Phi = f_c X_c + f_s X_s. \tag{4}$$

We can associate to the subcritical reactor a self sustained reactor having $k_{\infty,c}$ as multiplication factor. The spectrum φ_c is solution of

$$A \cdot \varphi_c = \frac{F \cdot \varphi_c}{k_{\infty,c}} X_c \tag{5}$$

and we normalize it in order to have

$$F \cdot \varphi_c = 1. \tag{6}$$

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http://dx.doi.org/10.1016/j.anucene.2014.12.006
0306-4549/

Thanks to the above normalization, Eq. (5) can be written as:

$$A \cdot \varphi_c = \frac{X_c}{k_{\infty,c}}. \quad (7)$$

We now consider a fictive self sustained reactor, differing from the previous one for having X_s as fission spectrum and $k_{\infty,s}$ as multiplication factor. Similarly we define for this reactor the spectrum φ_s normalized to have

$$F \cdot \varphi_s = 1 \quad (8)$$

and solution of

$$A \cdot \varphi_s = \frac{F \cdot \varphi_s}{k_{\infty,s}} X_s. \quad (9)$$

Combining Eqs. (7) and (9) and using the normalization in Eq. (8) we can write

$$A \cdot (f_c \varphi_c k_{\infty,c} + f_s \varphi_s k_{\infty,s}) = f_c X_c + f_s X_s, \quad (10)$$

which compared with Eq. (4) enables writing

$$\Phi = f_c \varphi_c k_{\infty,c} + f_s \varphi_s k_{\infty,s}. \quad (11)$$

Eq. (11) shows that the response to an external source of intensity f_s with spectrum X_s can be written as a combination of two self sustained spectra, of the actual and of a fictive reactor.

Multiplying scalar Eq. (11) by F and taking into account Eqs. (3), (6) and (8) we obtain the following equation

$$f_c = f_c k_{\infty,c} + f_s k_{\infty,s}, \quad (12)$$

which solved for f_c gives

$$f_c = \frac{f_s k_{\infty,s}}{1 - k_{\infty,c}}. \quad (13)$$

Now we can rewrite Eq. (11) using Eq. (13)

$$\Phi = f_s \left(\varphi_s + \frac{\varphi_c k_{\infty,c}}{1 - k_{\infty,c}} \right) k_{\infty,s} \quad (14)$$

and using the definition of reactivity

$$\varrho = \frac{k_{\infty,c} - 1}{k_{\infty,c}} \quad (15)$$

we obtain

$$\Phi = f_s \left(-\frac{\varphi_c}{\varrho} + \varphi_s \right) k_{\infty,s}. \quad (16)$$

We see that for a highly subcritical multiplying system ($\varrho \ll 0$) the spectrum tends toward φ_s, i.e. the self-sustained spectrum that would occur if the fission source had the spectrum of the external source. In case the external source has the same spectrum as the fission source, the shape of Φ is equal to φ_c. Obviously, for $\varrho \to 0$, the spectrum tends to the critical spectrum.

2.2. The subcritical multiplication factor

The definition of subcritical multiplication factor (Gandini, 2002; Talamo et al., 2012), transposed to the infinite medium reactor, is:

$$k_{\mathrm{src}} = \frac{F \cdot \Phi}{F \cdot \Phi + f_s} \quad (17)$$

which, taking into account Eq. (3), can be written as:

$$k_{\mathrm{src}} = \frac{f_c}{f_c + f_s}. \quad (18)$$

Using Eq. (13) we find the relationship between the definitions of multiplication factor in selfsustained mode and sustained by an external source:

$$k_{\mathrm{src}} = \frac{k_{\infty,s}}{1 - k_{\infty,c} + k_{\infty,s}}. \quad (19)$$

If the spectrum of the external source is equal to the fission spectrum, it follows that $k_{\infty,s} = k_{\infty,c}$, therefore, for the infinite reactor, $k_{\mathrm{src}} = k_{\infty,c}$.

2.3. The dependence of the multiplication factor on the fission spectrum

The dependence of the effective multiplication factor on the fission spectrum can be formulated using a perturbation theory approach. Let us define the problem adjoint to Eq. (9):

$$A^* \cdot \varphi_s^* = \frac{X_s \cdot \varphi_s^*}{k_{\infty,s}} F. \quad (20)$$

Multiplying Eq. (5) by φ_s^* and Eq. (20) by φ_c, subtracting, and taking into account that, by definition of adjoint operator

$$\varphi_c \cdot A^* \cdot \varphi_s^* = \varphi_s^* \cdot A \cdot \varphi_c \quad (21)$$

we obtain

$$\frac{X_c \cdot \varphi_s^*}{k_{\infty,c}} = \frac{X_s \cdot \varphi_s^*}{k_{\infty,s}}. \quad (22)$$

Since the adjoint flux does not depend on the fission spectrum (Dall'Osso and Brault, 2009), we remove the subscript s and rearrange Eq. (22) into the following form:

$$\frac{k_{\infty,s}}{k_{\infty,c}} = \frac{X_s \cdot \varphi^*}{X_c \cdot \varphi^*}. \quad (23)$$

We see that the k_∞ ratio of two reactor systems differing only by the fission emission spectrum is equal to the ratio of the importance of the neutrons emitted by the two fission sources. This property has been introduced by Spriggs et al. (2001) to compute the delayed neutron effectiveness factor using ratios of k-eigenvalues.

2.4. The gain factor

Introducing the definition of reactivity into Eq. (13) we obtain the expression of the gain factor (i.e. the ratio between the internal and external source)

$$\frac{f_c}{f_s} = -\frac{k_{\infty,s}}{\varrho k_{\infty,c}}. \quad (24)$$

We can observe that this definition differs from the classical one (gain factor = $-1/\varrho$), valid for monoenergetic neutrons, for the presence of the ratio $k_{\infty,s}/k_{\infty,c}$.

Taking into account Eq. (23), the expression of the gain factor in Eq. (24) becomes:

$$\frac{f_c}{f_s} = -\frac{X_s \cdot \varphi^*}{\varrho X_c \cdot \varphi^*} \quad (25)$$

which is similar to the formulation suggested by Spriggs et al. (1999).

2.5. The delayed neutron fission source

It is customary in the framework of reactor analyses to use the prompt spectrum instead of a spectrum combined of prompt (X_p) and delayed (X_d) neutrons, which would be more realistic:

$$X_c = (1 - \beta) X_p + \beta X_d. \quad (26)$$

Eq. (23) can be used to estimate the error committed on the multiplication factor by applying this approximation. If we replace in Eq. (23) X_s by X_p, $k_{\infty,s}$ by $k_{\infty,p}$ and using Eq. (26) we obtain