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The time dependence of the extinction probability with delayed neutrons

M.M.R. Williams [a], I. Pázsit [b,*]

[a] Imperial College London, Mechanical Engineering Department, Nuclear Engineering Group, Exhibition Road, London, SW7 2AZ, UK
[b] Department of Applied Physics, Division of Nuclear Engineering, Chalmers University of Technology, SE-41296 Göteborg, Sweden

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ABSTRACT

The calculation of the time-dependent extinction probability (EP) in an infinite medium of fissile material is calculated including delayed neutrons. This extension of the model reveals some interesting consequences, one of which is the appearance of an initial plateau in the EP in the range 10^{-6} to 10^{-4} s which is significantly less than the final asymptotic plateau value reached as time tends to infinity; i.e. the steady state EP. This steady state is delayed by the presence of the delayed neutrons by a time of the order of $3/\lambda$ ($\sim$36 s) where λ is the mean decay constant of the precursors. The presence of the initial plateau is due specifically to delayed neutrons and has not been commented on before. Several other situations are considered, with differing initial conditions, and are illustrated by numerical results.

1. Introduction

A large body of work exists on the extinction probability (EP) of a neutron chain reaction in a supercritical medium (Pázsit and Pál, 2008; Harris, 1963; Bell, 1963). Without exception, however, the quantitative influence of delayed neutrons has been neglected. That is not to say that the associated theory for including delayed neutrons in the EP has not been derived, but simply that no calculations have been carried out to study the practical significance of this effect. In this note we will set out the relevant equations for the EP, describe a method for solving them numerically and discuss the implications of the numerical results. For greater generality we will consider the time-dependent behaviour as well as the asymptotic limit for large times. It will be shown that a number of interesting and very practical consequences arise in the presence of delayed neutrons, especially when the reactivity is close to prompt critical. The calculations will be based on a point model system but the extension to a space-dependent problem is straightforward and follows the work of Williams (2004, 2008).

An important feature of the presence of delayed neutrons is the effect on the time behaviour of the EP. Subject to the initial state of one neutron and no precursors, this is seen to increase rapidly with a time constant associated with that of prompt neutrons and level off to a plateau. Normally, with no delayed neutrons, one would consider this plateau to be the asymptotic state and hence the maximum value of the EP. However, if we wait a further time, corresponding to about three times the average delayed neutron lifetime ($\sim$36 s), then the extinction probability increases again to a second and final plateau which is the desired asymptotic limit. Thus by ignoring delayed neutrons we fail to account for the initial EP which is substantially lower than the final asymptotic state and exists for about one second.

2. General theory

The essential theory for this problem may be found in the references cited above for the generating functions which fully define the associated probabilities of having n neutrons and n_i, $i = 1\ldots I$ delayed neutron precursors being present at a given time, due to either one starting neutron, or one delayed neutron precursor at time $t = 0$. Let us briefly discuss the background theory by first defining the probability

$$p[n(t) = n, n_1(t) = n_1, n_2(t) = n_2, \ldots n_I(t) = n_I | n(0) = 1, \\ n_1(t) = 0, n_2(t) = 0, \ldots n_I(t) = 0] \tag{1}$$

This assumes that initially, at $t = 0$, there is one neutron in the system and no precursors. Similarly,

$$p[n(t) = n, n_1(t) = n_1, n_2(t) = n_2, \ldots n_I(t) = n_I | n(0) = 0, \\ n_1(t) = 1, n_2(t) = 0, \ldots n_I(t) = 0] \tag{2}$$

* Corresponding author.
E-mail addresses: mmrw@btinternet.com (M.M.R. Williams), imre@chalmers.se (I. Pázsit).

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assumes that initially, at $t = 0$, there are no neutrons in the system and one precursor of type 1. Various initial conditions may be defined in this way. The above notations can be shortened to $p(n, n_1, \ldots n_I|1, 0, \ldots 0)$, etc.

A backward-type master equation for the $p(n, n_1, \ldots n_I|1, 0, \ldots 0)$ can be derived as follows (Pázsit and Pál, 2008):

$$p(n,n_1,\ldots n_I|1,0,\ldots 0) = \delta_{n,1}\delta_{n_1,0}\ldots\delta_{n_I,0}e^{-\lambda_t t} + \lambda_t\int_0^t dt' e^{-\lambda_t(t-t')}\sum_{k,k_1,\ldots k_I} f(k,k_1,\ldots k_I) \times \sum_{\substack{n^{(0)}+n^{(1)}+\ldots n^{(I)}=n \\ n_1^{(0)}+n_1^{(1)}+\ldots n_1^{(I)}=n_1 \\ \ldots \\ n_I^{(0)}+n_I^{(1)}+\ldots n_I^{(I)}=n_I}}\prod_{i=0}^{I}A_{k_i}^{(i)}(n^{(i)},n_1^{(i)}\ldots n_I^{(i)})$$

where the quantities $A_{k_i}^{(i)}$ are defined as

$$A_k^{(0)}(n^{(0)},n_1^{(0)}\ldots n_I^{(0)}) = \sum_{\substack{n_{10}^{(0)}+n_{20}^{(0)}+\ldots n_{k0}^{(0)}=n^{(0)} \\ n_{11}^{(0)}+n_{21}^{(0)}+\ldots n_{k1}^{(0)}=n_1^{(0)} \\ \ldots \\ n_{1I}^{(0)}+n_{2I}^{(0)}+\ldots n_{kI}^{(0)}=n_I^{(0)}}}\prod_{j=1}^{k}p(n_{j0}^{(0)},n_{j1}^{(0)},\ldots n_{jI}^{(0)}|1,0\ldots 0)$$

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$$A_{k_I}^{(I)}(n^{(I)},n_1^{(I)}\ldots n_I^{(I)}) = \sum_{\substack{n_{10}^{(I)}+n_{20}^{(I)}+\ldots n_{k_I0}^{(I)}=n^{(I)} \\ n_{11}^{(I)}+n_{21}^{(I)}+\ldots n_{k_I1}^{(I)}=n_1^{(I)} \\ \ldots \\ n_{1I}^{(I)}+n_{2I}^{(I)}+\ldots n_{k_II}^{(I)}=n_I^{(I)}}}\prod_{j=1}^{k_I}p(n_{j0}^{(I)},n_{j1}^{(I)},\ldots n_{jI}^{(I)}|0,0\ldots 1)$$

where the definitions of $n_0 \equiv n$ and $k_0 \equiv k$ are used when applicable.

The equations for the $p(n, n_1, \ldots n_I|0, 1, \ldots 0)$ etc., i.e. when the process was started by having one single delayed neutron precursor in the system, read as

$$p(n,n_1,\ldots n_I,t|0,1,0,..0) = \delta_{n,0}\delta_{n_1,1}\delta_{n_2,0}\ldots\delta_{n_I,0}e^{-\lambda_1 t} + \lambda_1\int_0^t dt' e^{-\lambda_1(t-t')}p(n,n_1,\ldots n_I,t'|1,0,0,..0)$$

$$p(n,n_1,\ldots n_I,t|0,0,1,..0) = \delta_{n,0}\delta_{n_1,0}\delta_{n_2,1}\ldots\delta_{n_I,0}e^{-\lambda_2 t} + \lambda_2\int_0^t dt' e^{-\lambda_2(t-t')}p(n,n_1,\ldots n_I,t'|1,0,0,..0)$$

etc

$$p(n,n_1,\ldots n_I,t|0,0,0,..1) = \delta_{n,0}\delta_{n_1,0}\delta_{n_2,0}\ldots\delta_{n_I,1}e^{-\lambda_I t} + \lambda_I\int_0^t dt' e^{-\lambda_I(t-t')}p(n,n_1,\ldots n_I,t'|1,0,0,..0)$$

The generating function associated with the above probabilities is

$$g(z,z_1,\ldots z_I,t|1,0,\ldots 0) = \sum_{n=0}^{\infty}\sum_{n_1=0}^{\infty}\ldots\sum_{n_I=0}^{\infty}z^n z_1^{n_1}\ldots z_I^{n_I}p[n(t) = n, n_1(t) = n_1, n_2(t) = n_2,\ldots n_I(t) = n_I|1,0,\ldots 0] \tag{3}$$

In Pázsit and Pál (2008) it is shown that the balance equation for this generating function is

$$g(z,z_1,\ldots z_I,t|1,0,\ldots 0) = ze^{-\lambda_t t} + \lambda_t\int_0^t dt' e^{-\lambda_t(t-t')}q[g(z,z_1,\ldots z_I,t'|1,0,\ldots 0), g(z,z_1,\ldots z_I,t'|0,1,\ldots 0),\ldots g(z,z_1,\ldots z_I,t'|0,0,\ldots 1)] \tag{4}$$

where λ_t is the total interaction intensity including absorption, fission and scattering, and

$$\begin{aligned} &g(z,z_1,\ldots z_I,t|0,1,0,..0) = z_1 e^{-\lambda_1 t} + \lambda_1\int_0^t dt' e^{-\lambda_1(t-t')} g(z,z_1,\ldots z_I,t'|1,0,0,\ldots 0) \\ &g(z,z_1,\ldots z_I,t|0,0,1,..0) = z_2 e^{-\lambda_2 t} + \lambda_2\int_0^t dt' e^{-\lambda_2(t-t')} g(z,z_1,\ldots z_I,t'|1,0,0,\ldots 0) \\ &etc \\ &g(z,z_1,\ldots z_I,t|0,0,0,..1) = z_I e^{-\lambda_I t} + \lambda_I\int_0^t dt' e^{-\lambda_I(t-t')} g(z,z_1,\ldots z_I,t'|1,0,0,\ldots 0) \end{aligned} \tag{5}$$

where λ_i are the delayed neutron precursor decay constants and the generating function $q[z, z_1, \ldots z_I]$ is given by

$$q[z,z_1,\ldots z_I] = \sum_{k=0}^{\infty}\sum_{k_1=0}^{\infty}\ldots\sum_{k_I=0}^{\infty}z^k z_1^{k_1}\ldots z_I^{k_I}f(k,k_1,\ldots k_I) \tag{6}$$

with $f(k, k_1, \ldots k_I)$ being the probability that k prompt neutrons and $k_1, k_2, \ldots k_I$ delayed neutron precursors are born in one reaction. We will discuss the general form of $f(k, k_1, \ldots k_I)$ below. The notation will be simplified as follows

$$G_{d_i}(z,\mathbf{z},t) = g(z,z_1,\ldots z_I,t|0,i) \quad \text{and} \quad G(z,\mathbf{z},t) = g(z,z_1,\ldots z_I,t|1,0,0,\ldots 0) \tag{7}$$

where the argument i in the delayed neutron symbol denotes the position of 1 in the conditional statements.

For a quantitative study, the statistics of the branching, expressed by the generating function $q[z, z_1, \ldots z_I]$ has to be specified. Since in an energy independent description scattering events disappear from the probability balance equations, we will consider that the neutrons can only undergo fission or capture with the corresponding intensities λ_f and λ_c, with $\lambda_a = \lambda_f + \lambda_c$ being the total reaction intensity. However, in order to show how this disappearance arises, and for consistency, we include scattering in the general derivation. The number distribution of neutrons and delayed neutron precursors in one *fission* event is described by the distribution $p(k, k_1, k_2, \ldots k_I)$. On physical grounds and in the absence of evidence to the contrary, it is customary to assume that the generation of prompt and delayed neutrons are independent processes, i.e. that $p(k, k_1, k_2, \ldots k_I)$ factorises into the product $p^{(0)}(k)p^{(1)}(k_1)\ldots p^{(I)}(k_I)$. The relationship between the two distributions is then given as

$$f(k,k_1,\ldots k_I) = \frac{\lambda_c}{\lambda_t}\delta_{k,0}\delta_{k_1,0}\ldots\delta_{k_I,0} + \frac{\lambda_s}{\lambda_t}\delta_{k,1}\delta_{k_1,0}\ldots\delta_{k_I,0} + \frac{\lambda_f}{\lambda_t}p^{(0)}(k)p^{(1)}(k_1)\ldots p^{(I)}(k_I) \tag{8}$$

Eq. (8) shows that the distribution of prompt and delayed particles born *per reaction* is not factorisable, i.e. the underlying processes are not independent. Physically, the reason is that although in fission the prompt neutron and delayed neutron generation are independent processes, the same is not true for capture and scattering since for capture it means that zero prompt neutron emission will always be accompanied by zero precursor generation, and for scattering it means that the re-emergence of one neutron from the collision will always be accompanied by zero precursor generation. This creates a correlation between the production of the two particle types. Inserting $f(\ldots)$ into the definition of the generating function leads to

$$q[G,G_{d1},G_{d2},\ldots G_{dI}] = \frac{\lambda_c}{\lambda_t} + \frac{\lambda_s}{\lambda_t}G + \frac{\lambda_f}{\lambda_t}\sum_{k=0}^{\infty}\sum_{k_1=0}^{\infty}\ldots\sum_{k_I=0}^{\infty}G^k G_{d1}^{k_1}\ldots G_{dI}^{k_I}p^{(0)}(k)p^{(1)}(k_1)p^{(2)}(k_2)\ldots p^{(I)}(k_I) \tag{9}$$

or more simply