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Explicit appropriate basis function method for numerical solution of stiff systems

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ABSTRACT

In this paper, an explicit numerical method, called the appropriate basis function method, is presented. The explicit appropriate basis function method differs from the power series method because it employs an appropriate basis function such as the exponential function, or periodic function, other than a polynomial, to obtain approximate numerical solutions. The method is successful and effective for the numerical solution of the first order ordinary differential equations. Two examples are presented to show the ability of the method for dealing with linear and nonlinear systems of differential equations.

1. Introduction

Numerous works have been focusing on the development of more advanced and efficient methods for solving ordinary differential equations (ODEs) (Butcher, 2000; Hajmohammadi et al., 2012, 2014a). For example, Wazwaz (2010) presented a new algorithm for solving in ODE's of integro–differential type, and Hajmohammadi and Nourazar (2014b, c) presented a new algorithm for solving in ODE's of eigen-value type based on semi-analytical method and pure-analytical method for solving in ODE's of non-linear type, respectively. Implicit numerical methods are usually used for solving ordinary differential equations, in particular for some stiff problems, in which the implicitness will cause the method very involved compared to explicit methods (Li et al., 2009; Zhu et al., 2012). Indeed, explicit numerical methods require smaller step sizes in situations where implicit methods would not. It should be emphasized that explicit methods also show important advantages, some authors have developed many explicit methods for stiff systems (Guzel and Bayram, 2005).

There has been a great deal of research that focused on eliminating the stiffness problem of reactor kinetics (Koclas et al., 1996; Chen et al., 2013). And there are several methods especially adapted for solving the initial value problems for stiff systems of ordinary differential equations (Aboanber and Hamada, 2003; Aboanber, 2004; Tashakor et al., 2010). Among the methods are numerical integration using Simpson's rule, finite element method, Runge–Kutta procedures, quasi-static method, piecewise polynomial approach and other methods (Li et al., 2010; Abdallah and Nahla, 2011; Hamada, 2013). Most of these methods are successful in some specific problems, but still suffer, more or less, from disadvantages as mentioned by Chen et al., 2013.

In this paper, an explicit numerical method for stiff systems is developed and tested. The method differs from the power series method (Guzel and Bayram, 2005) in the idea that employs an appropriate basis function, such as the exponential function, periodic function, and so on, other than a polynomial to obtain approximate numerical solutions. In the other hand, the method is considerably more accurate than the power series method, as demonstrated in following sections.

2. Appropriate basis function method

2.1. Basic ideas

Since every ordinary differential equation with order n can be written as a system consisting of n ordinary differential equations with order one, our study is restricted to a system of first order differential equations, which can be written as follows

$$\begin{aligned} &y' = f(t,y),\ t \in [0,T],\ \|y\| < +\infty, \\ &y(0) = y_0, \end{aligned} \tag{1}$$

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whose theoretical solution is $y(t)$. Let y_n be an approximation to $y(t_n)$. Here

$$y = [y_1, y_2, \ldots, y_k]^T,$$

$$f = [f_1, f_2, \ldots, f_k]^T,$$

and

$$y_0 = [y_{01}, y_{02}, \ldots, y_{0k}]^T.$$

It is assumed that f and y are sufficient differentiability (Guzel and Bayram, 2005). The power series method (Guzel and Bayram, 2005) assumed that the solution of Eq. (1) can be expressed by a polynomial expression as follows

$$y = \sum_{i=0}^{\infty} A_i x^i, \tag{2}$$

where A_i is a vector function which is the same size as y_0.

However, the authors think that it should choose an appropriate function as the basis function according to the characteristic of stiff systems in order to improve the computational efficiency and accuracy.

It is well known that a real function can be expressed by some different basis functions. For example, let us consider $u(t) = \tanh(t)$, which can be expressed by a polynomial expression as follows

$$\tanh(t) \approx t - \frac{t^3}{3} + \frac{2t^5}{15} - \frac{17t^7}{315} + \cdots, \tag{3}$$

Then Eq. (3) converges to the exact solution $u(t)$ only in a small region $0 \leqslant t < 0.5$ (see Fig. 1). However, by means of the exponential function as the basis function, one has

$$\tanh(t) \approx 1 + \lim_{m\to+\infty}\left[2\sum_{n=1}^{m}(-1)^n e^{-2nt} + (-1)^{m+1} e^{-(2m+1)t}\right], \tag{4}$$

which converges to the exact solution $u(t)$ in the whole region $0 \leqslant t < +\infty$ (Liao and Tan, 2007). And, even taking the first few terms, Eq. (4) can also give the very accurate approximation. For example, when $m = 2$, Eq. (4) can be written as follows

$$\tanh(t) \approx 1 - 2e^{-2t} + 2e^{-4t} - e^{-5t},$$

which agrees well with the exact solution $\tanh(t)$ in the whole region $0 \leqslant t < +\infty$. In addition, the numerical tests in Table 1 demonstrate that the efficiency of the exponential function is better than that of a polynomial expression. When the approximation solution is expressed by the exponential function $u(t)$, it will take a computing time of 0.094 s to obtain the value of $u(t)$ in the time interval [0, 3/2] with the time step $h = 0.00001$ s. However, using a polynomial as the basis function, it will take a computing time of 0.203 to 0.297 s.

Therefore, one can get the best approximation by means of an appropriate basis function. For example, a periodic solution is expressed more efficiently by periodic basis functions than by a polynomial expression. The solution of Eq. (1) can be expressed by a set of appropriate basis functions $\{\omega_i(i)|i \geq 0\}$ as follows

$$y = \sum_{i=0}^{\infty} A_i \omega_i(x), \tag{5}$$

where $\omega_i(x)$ is the ith term of the basis functions. For example, if the set of appropriate basis functions is $\{\exp(ix)|i \geq 0\}$, $\omega_i(x)$ will be $\exp(ix)$.

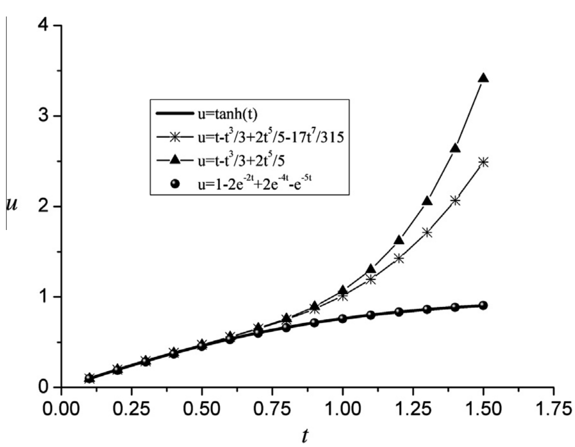

Fig. 1. Exact and approximation solutions of $u(t) = \tanh(t)$.

2.2. Scheme of the method

The way that the appropriate basis function method is used in practice is carried out by the following computation steps:

Step 1: Choosing the order of approximation solution, m.
Step 2: When $x = 0$, according to Eq. (5), we can get

$$y^{(n)}(0) = \sum_{i=0}^{m} A_i \omega_i^{(n)}(0). \tag{6}$$

Step 3: According to Eq. (1) and the initial conditions, the values or expressions of $y(0), y'(0), y''(0), \ldots, y^{(m)}(0)$ can be obtained by the iterative computation.
Step 4: When $n = 0,\ 1,\ 2, \ldots,\ m$, according to Eq. (6), the linear equations of A_i can be derived, and it is easy for us to develop a computer code to calculate the coefficient A_i. Then, substituting A_i into Eq. (5), the m th-order approximation solution of Eq. (1) is obtained .
Step 5: Making step size of x to be h and substituting it into the mth-order approximation solution of Eq. (1), we have y at $x = x_0 + h$.
Step 6: Repeating above steps 1 to 5, we can obtain the numerical solution of Eq. (1).

3. Numerical experiments

The appropriate basis function method and the power series method (Guzel and Bayram, 2005) are compared in the following cases. We arranged the independent variable x, the step size h,

Table 1
Comparison of approximation solutions of $u(t) = \tanh(t)$ in the time interval [0, 3/2] by using $h = 10^{-5}$.

Type of approximation	Solutions for different time							Computing time (s)
	$t = 0.8$	$t = 1$	$t = 1.1$	$t = 1.2$	$t = 1.3$	$t = 1.4$	$t = 1.5$	
$u(t) = t - \frac{t^3}{3} + \frac{2t^5}{5} - \frac{17t^7}{315}$	0.7491	1.0127	1.1954	1.4260	1.7142	2.0677	2.4904	0.297
$u(t) = t - \frac{t^3}{3} + \frac{2t^5}{5}$	0.7604	1.0667	1.3005	1.6193	2.0528	2.6366	3.4125	0.203
$u(t) = 1 - 2e^{-2t} + 2e^{-4t} - e^{-5t}$	0.6594	0.7592	0.7989	0.8326	0.8610	0.8849	0.9048	0.094
Accurate solution	$t = 0.8$	$t = 1$	$t = 1.1$	$t = 1.2$	$t = 1.3$	$t = 1.4$	$t = 1.5$	
$u(t) = \tanh(t)$	0.6640	0.7616	0.8005	0.8337	0.8617	0.8854	0.9052	