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A fission collision separation method for efficient incident flux response expansion coefficient generation

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ABSTRACT

A new fission collision (generation) separation method has been developed for generating incident flux response expansion coefficients and implemented into the hybrid stochastic/deterministic coarse mesh transport (COMET) method for whole core criticality (eigenvalue) calculations. During the stochastic pre-computation of the response library, all coefficients associated with each fission neutron generation are tallied separately. This enables COMET to directly calculate the eigenvalue (k) dependent response coefficients on-the-fly as a superposition of contributions from the n^{th} fission neutron generation scaled by the factor $\left(\frac{1}{k}\right)^n$. The new method is as accurate as or more accurate and significantly more efficient than the existing response methods, regardless of whether the fission source is treated explicitly or implicitly as in COMET. The method also removes the eigenvalue range restriction on the response library.

The 3D C5G7 benchmark problem was used to test the accuracy and efficiency of the new response coefficient generation method at both lattice (local) and core (global) levels. For lattice calculations, the response coefficients computed by the new method were compared with those computed by the direct Monte Carlo method. It was found that the truncation errors of the fission neutron separation method at expansion order 3 are much lower than the stochastic uncertainties of the response coefficients and consequently can be ignored. The average truncation errors of the new method ranged from 0% to 0.004% and 0% to 0.08% for the surface-to-surface and fission density response coefficients, respectively. For core level calculations, the COMET whole-core solutions based on the response coefficient libraries generated by the new method and the interpolation method were compared with direct (reference) Monte Carlo calculations using the MCNP code. It was found that the error in the eigenvalues predicted by COMET based on the new method is within 11–26 pcm for three core configurations with and without control rods. These eigenvalue results are about 15 pcm more accurate than those based on the interpolation method, although still within the overall uncertainty. The corresponding assembly and pin fission densities are almost identical for the two methods. These comparisons indicate that the new method is as accurate as or slightly more accurate than the interpolation method, depending on the comparison parameter. The new method was found to be about 5 times faster than the interpolation method for the generation of the response function library.

1. Introduction

The coarse mesh transport (COMET) (Mosher and Rahnema, 2006; Zhang and Rahnema, 2012a) method has been developed to provide whole core transport analyses of highly heterogeneous nuclear reactor cores. COMET is based solely on transport theory and does not require spatial homogenization. Because of its high fidelity and very fast computational speed, COMET is ideal for performing neutronic analysis in highly heterogeneous and advanced reactor cores in which diffusion approximation is inadequate. The COMET method is started with dividing the spatial domain of a global problem into a number of coarse meshes. By expanding the angular flux on the coarse mesh interfaces, the exiting flux from a coarse mesh can be iteratively calculated as a superposition of the contributions due to incoming fluxes from all contiguous coarse meshes via pre-computed response expansion coefficients. Extensive benchmark calculations (Zhang and Rahnema, 2012b,c,d) against MCNP (Briesmeister, 1997) in various reactor cores such as PWRs, BWRs, CANDUs, HTTRs have shown that its accuracy is close to the Monte Carlo method, while it is highly efficient (orders of magnitude faster than Monte Carlo) in performing whole core calculations given a pre-computed (incident flux)

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response (expansion coefficient) library. It should be noted that the response expansion coefficients are a function of coarse mesh types (configuration and material) as well the core eigenvalue k.

In this paper, a fission collision separation (FCS) method is developed to significantly improve the computational efficiency in generating the eigenvalue-dependent response library. The remainder of the paper is organized as follows. The new response coefficient generation method is introduced in Section 2. The numerical results are presented and discussed in Section 4. Concluding remarks are found in Section 4.

2. Fission collision separation (FCS) method for response coefficient generation

The COMET method consists of 4 numerical steps: decomposition of the spatial domain into non-overlapping coarse meshes, local calculations to generate response coefficients for each unique coarse mesh, global calculations to converge on the core eigenvalue and incoming/outgoing flux moments crossing mesh interfaces, and construction of the flux and/or fission density distribution within each coarse mesh.

After the spatial domain of a global eigenvalue problem is divided into a number of non-overlapping coarse meshes $\{V_1, V_2, \ldots\}$ and the fluxes on mesh interfaces are expanded in term of orthogonal expansion functions, the outgoing current moment from a coarse mesh can be calculated as a superposition of the contributions responding to incoming currents entering from all contiguous coarse meshes:

$$J_{is}^{+,m} = \sum_{s',m'} J_{is'}^{-,m'} R_{s's}^{m'm}. \tag{1}$$

Here, $J_{is}^{\pm,m}$ are the m^{th} moment of the outgoing/incoming current crossing surface ∂V_{is}, and $R_{s's}^{m'm}$ represents surface-to-surface response coefficient which has the following relation with the surface-to-volume response function $R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E\right)$.

$$R_{s's}^{m'm} = \int_{\partial V_{is}} d\vec{r} \int_{\hat{n}_{is}^{+}\cdot\widehat{\Omega}>0} d\widehat{\Omega} \int dE \left(\hat{n}_{is}^{+}\cdot\widehat{\Omega}\right)\Gamma_m\left(\vec{r},\widehat{\Omega},E\right) R_{is'}^{m'}(\vec{r},\widehat{\Omega},E). \tag{2}$$

Essentially, the surface-to-volume response function $R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E\right)$ represents the flux distribution within a coarse mesh i responding to a unit incident current impinging on the coarse mesh boundary $\partial V_{is'}$, and it is the solution to the following local fixed-source transport problem:

$$\mathbf{H}R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E\right) = \frac{1}{k}\mathbf{F}R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E\right) \text{ for } \vec{r}\in V_i. \tag{3}$$

with boundary condition:

$$R_{is'}^{m'}(\vec{r},\widehat{\Omega},E) = \begin{cases} \Gamma_{m'}(\vec{r},\widehat{\Omega},E) & \vec{r}\in\partial V_{is'} \text{ and } \widehat{\Omega}\cdot\hat{n}_{is'}^{+}<0 \\ 0 & \vec{r}\in\partial V_i-\partial V_{is'} \text{ and } \widehat{\Omega}\cdot\hat{n}_{i}^{+}<0 \end{cases}. \tag{4}$$

Here, $\hat{n}_{is}^{+}$ denotes the outward normal on ∂V_{is}, $\Gamma_m(\vec{r},\widehat{\Omega},E)$ are the expansion functions, k is the global (core) eigenvalue, and the operators $\mathbf{H}$ and $\mathbf{F}$ are defined below:

$$\mathbf{H} = \widehat{\Omega}\cdot\nabla + \sigma_t(\vec{r},E) - \int dE' \int_{4\pi} d\widehat{\Omega}'\sigma_s\left(\vec{r},\widehat{\Omega}',E'\to\widehat{\Omega},E\right), \tag{5}$$

$$\mathbf{F} = \frac{1}{4\pi}\chi(\vec{r},E)\int dE' \int_{4\pi} d\widehat{\Omega}'\nu\sigma_f(\vec{r},E'), \tag{6}$$

where σ_t, σ_s and σ_f are the macroscopic total, scattering and fission cross sections, respectively.

It should be noted that Eq. (3) is a fixed-source problem with the fission neutron source scaled by $1/k$. It is obvious that response coefficients depend not only on the coarse mesh type, but also on the core eigenvalue k. Since k is not a known *priori*, two approaches have been previously developed to calculate the response coefficients at an arbitrary eigenvalue. In the first approach, the response coefficient library is first generated at a set of pre-defined eigenvalues, and its value at an arbitrary eigenvalue k is computed by interpolation. This approach is straightforward, but it requires generating response coefficients at more than one eigenvalue points. In the second approach (Zhang and Rahnema, 2014), response functions are first computed at a reference eigenvalue, and then their value at any given k is calculated as a perturbation to the reference case. The perturbation approach is shown to be more efficient than the interpolation method, but it requires solving a set of adjoint problems that could be difficult to implement.

In this work, the response functions are separated into the 0^{th}, 1^{st}, 2^{nd}, and higher collided fission neutrons. The 0^{th} fission collision response function $R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;0\right)$, which represents the neutron distribution corresponding to the incoming flux $\Gamma_{m'}\left(\vec{r},\widehat{\Omega},E\right)$ without making fission collisions within coarse mesh i, is the solution to the following transport problem,

$$\mathbf{H}R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;0\right) = 0 \text{ for } \vec{r}\in V_i, \tag{8}$$

with boundary condition

$$R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;0\right) = \begin{cases} \Gamma_{m'}\left(\vec{r},\widehat{\Omega},E\right) & \vec{r}\in\partial V_{is'} \text{ and } \widehat{\Omega}\cdot\hat{n}_{is'}^{+}<0 \\ 0 & \vec{r}\in\partial V_i-\partial V_{is'} \text{ and } \widehat{\Omega}\cdot\hat{n}_{is'}^{+}<0 \end{cases}. \tag{9}$$

Similarly, the n^{th} $(n>0)$ fission neutron response function $R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;n\right)$, which represents the neutron distribution corresponding to the incoming flux $\Gamma_{m'}\left(\vec{r},\widehat{\Omega},E\right)$ making the n^{th} fission collisions within coarse mesh i, satisfies the following recurrence relation

$$\mathbf{H}R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;n\right) = \mathbf{F}R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;n-1\right) \text{ for } \vec{r}\in V_i, \tag{8}$$

with boundary condition

$$R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;n\right) = 0, \qquad \vec{r}\in\partial V_i \textit{ and } \widehat{\Omega}\cdot\hat{n}_{i}^{+}<0 \tag{9}$$

Clearly, the surface-to-volume response function $R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E\right)$ can be computed as the following superposition.

$$R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E\right) = \sum_{n=0}^{\infty}\frac{1}{k^n}R_{is'}^{m'}\left(\vec{r},\widehat{\Omega},E;n\right). \tag{10}$$

Similarly, the surface-to-surface response coefficient $R_{s's}^{m'm}$ and the fission density response coefficient $RF_{s'}^{m',j}$ can be computed as

$$R_{s's}^{m'm} = \sum_{n=0}^{\infty}\frac{1}{k^n}R_{s's}^{m'm}(n), \tag{11}$$

and

$$RF_{s'}^{m',j} = \sum_{n=0}^{\infty}\frac{1}{k^n}RFn_{s'}^{m',j}(n), \tag{12}$$

where, $R_{s's}^{m'm}(n)$ and $RF_{s'}^{m',j}(n)$ are defined as

$$R_{s's}^{m'm}(n) = \int_{\partial V_{is}} d\vec{r}\int_{\hat{n}_{is}^{+}\cdot\widehat{\Omega}>0} d\widehat{\Omega}\int dE\left(\hat{n}_{is}^{+}\cdot\widehat{\Omega}\right)\Gamma_m\left(\vec{r},\widehat{\Omega},E\right)R_{is'}^{m'}(\vec{r},\widehat{\Omega},E;n), \tag{13}$$

$$RF_{s'}^{m',j}(n) = \int_{V_i^j} d\vec{r}\int_{4\pi} d\widehat{\Omega}\int dE\nu\sigma_f(\vec{r},E)Rn_{is'}^{m'}(\vec{r},\widehat{\Omega},E;n). \tag{14}$$

It should be pointed out that the new method is very easy to implement into a stochastic response function generator. For each response function calculation, the position, direction and energy of source particles are sampled from the incident flux phase space distribution $\Gamma_{m'}\left(\vec{r},\widehat{\Omega},E\right)$ imposed on the coarse mesh boundary $\partial V_{is'}$ and then particle histories are tracked until they are terminated by