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Monte Carlo methods for reactor period calculations

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ABSTRACT

Several technological issues, such as reactor start-up analysis or kinetics studies of accelerator-driven systems, demand the asymptotic time behaviour of neutron transport to be assessed. Typically, this amounts to solving an eigenvalue equation associated to the Boltzmann operator, whose precise nature depends on whether delayed neutrons are taken into account. The inverse of the dominant eigenvalue can be physically interpreted as the asymptotic reactor period. In this work, we propose a Monte Carlo method for determining the dominant alpha eigenvalue of the Boltzmann operator and the associated fundamental mode for arbitrary geometries, materials, and boundary conditions. Extensive verification tests of the algorithm are performed, and Monte Carlo calculations are finally validated against reactor period measurements carried out at the ORPHEE facility of CEA/Saclay.

1. Introduction

In many technological applications, encompassing reactor start-up analysis, reactivity measurements and accelerator-driven systems, one is interested in determining the time behaviour of the neutron flux φ in a system, starting from a given initial condition (Bell and Glasstone, 1970; Pázsit and Pál, 2008; Pfeiffer et al., 1974; Hansen, 1985; Cao and Lee, 2008; Persson, 2008; Keepin, 1965). The full description of such behaviour is provided by the time-dependent Boltzmann equation, possibly coupled with the equations for the precursors concentrations $c_{i,j}$, which read (Bell and Glasstone, 1970)

$$\frac{1}{\mathrm{v}}\frac{\partial}{\partial t}\varphi(\mathbf{r},\mathbf{v},t)+L\varphi(\mathbf{r},\mathbf{v},t)=F_p\varphi(\mathbf{r},\mathbf{v},t)+\sum_{i,j}\chi_d^{i,j}(\mathbf{r},v)\lambda_{i,j}c_{i,j}(\mathbf{r},t) \tag{1}$$

and

$$\frac{\partial}{\partial t}c_{i,j}(\mathbf{r},t)=\int \nu_d^{i,j}(\mathrm{v}')\Sigma_f^i(\mathbf{r},\mathrm{v}')\varphi(\mathbf{r},\mathbf{v}',t)\,\mathrm{d}\mathbf{v}'-\lambda_{i,j}c_{i,j}(\mathbf{r},t). \tag{2}$$

We have here defined the net disappearance operator

$$Lf=\boldsymbol{\Omega}\cdot\nabla f+\Sigma_t f-\int \Sigma_s(\mathbf{r},\mathbf{v}'\to\mathbf{v})f(\mathbf{r},\mathbf{v}')\,\mathrm{d}\mathbf{v}', \tag{3}$$

and the prompt fission operator

$$F_p f=\chi_p(\mathbf{r},\mathrm{v})\int \nu_p(\mathrm{v}')\Sigma_f(\mathbf{r},\mathrm{v}')f(\mathbf{r},\mathbf{v}')\,\mathrm{d}\mathbf{v}'. \tag{4}$$

Notation is as follows: v is the neutron speed, $\mathbf{r}$ is the position vector and $\boldsymbol{\Omega}$ is the angular direction vector, Σ_t is the total cross-section, Σ_s is the differential scattering cross-section, χ_p is the normalized spectrum for prompt fission neutrons, ν_p is the average number of prompt fission neutrons, Σ_f is the fission cross-section, $\chi_d^{i,j}$ is the normalized spectrum of delayed neutrons emitted from precursor family j of isotope i, $\lambda_{i,j}$ is the decay constant of precursor family j of isotope i, $\nu_d^{i,j}$ is the average number of delayed fission neutrons of precursor family j of isotope i, and the double sum is extended over all fissile isotopes i and over all precursor families j for each fissile isotope. The equations above are completed by assigning the proper initial and boundary conditions for φ and $c_{i,j}$. We have assumed here that all physical parameters (such as cross-sections, velocity spectra, and so on) are time-independent: this amounts to taking t shorter than the typical time scale of thermal–hydraulic and Doppler feedback (Keepin, 1965; Akcasu et al., 1971). If N fissile isotopes are present, each associated to M precursors families, Eqs. (1) and (2) form a system of $1+N\times M$ equations to be solved simultaneously.

Often, only the long-time (asymptotic) behaviour is required so as to characterize the system evolution (Bell and Glasstone, 1970; Duderstadt and Martin, 1979). Then, for bounded domains an exponential relaxation of the kind

$$\varphi(\mathbf{r},\mathbf{v},t)=\sum_{\alpha}\varphi_\alpha(\mathbf{r},\mathbf{v})e^{\alpha t} \tag{5}$$

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and

$$c_{i,j}(\mathbf{r},t)=\sum_{\alpha}c_{\alpha}^{i,j}(\mathbf{r})e^{\alpha t} \tag{6}$$

is postulated for both the neutron flux and the precursors concentrations, where the values αs represent the relaxation frequencies (Bell and Glasstone, 1970; Duderstadt and Martin, 1979). Eqs. (5) and (6) formally stem from imposing the separation of variables in Eqs. (1) and (2), and can be more rigorously justified by resorting to Laplace transform or equivalently to spectral analysis (Duderstadt and Martin, 1979). Yet, proving the feasibility of such an expansion is highly non-trivial in general, and precise (although not very restrictive) conditions are required on the geometry of the domain and on the material cross-sections (Bell and Glasstone, 1970; Duderstadt and Martin, 1979; Larsen and Zweifel, 1974). Here, for the sake of simplicity, we will assume that such conditions are met (which is typically the case for almost all systems of practical interest) and that there exists a set of values α such that separation of variables is allowed.

Then, substituting Eqs. (5) and (6) into Eqs. (1) and (2), respectively, yields the so-called (coupled) alpha-static equations

$$\frac{\alpha}{\mathrm{v}}\varphi_{\alpha}(\mathbf{r},\mathbf{v})+L\varphi_{\alpha}(\mathbf{r},\mathbf{v})=F_p\varphi_{\alpha}(\mathbf{r},\mathbf{v})+\sum_{i,j}\chi_d^{i,j}(\mathbf{r},\mathrm{v})\lambda_{i,j}c_{\alpha}^{i,j}(\mathbf{r}) \tag{7}$$

and

$$\alpha c_{\alpha}^{i,j}(\mathbf{r})=\int \nu_d^{i,j}(\mathrm{v}')\Sigma_f^{i}(\mathbf{r},\mathrm{v}')\varphi_{\alpha}(\mathbf{r},\mathbf{v}')\,\mathrm{d}\mathbf{v}'-\lambda_{i,j}c_{\alpha}^{i,j}(\mathbf{r}), \tag{8}$$

which formally represent a system of eigenvalue equations for the flux φ_{α} and the precursors $c_{\alpha}^{i,j}$, the eigenvalues being α. In principle, Eqs. (7) and (8) have $1+N\times M$ sets of eigenvalues associated to the prompt and delayed components (Bell and Glasstone, 1970; Cohen, 1958; Henry, 1964).

In order to determine the asymptotic time behaviour of the system, the algebraically largest eigenvalue α must be found, so that the corresponding fundamental mode $\varphi_{\alpha}(\mathbf{r},\mathbf{v})$ will provide the space and velocity shape of the neutron flux at long times. For short observation times t, the impact of delayed neutrons can be safely neglected (i.e., $c_{\alpha}^{i,j}=0$), so that Eq. (7) yields

$$\frac{\alpha}{\mathrm{v}}\varphi_{\alpha}(\mathbf{r},\mathbf{v})+L\varphi_{\alpha}(\mathbf{r},\mathbf{v})=F_p\varphi_{\alpha}(\mathbf{r},\mathbf{v}), \tag{9}$$

where α takes the name of prompt time eigenvalue and physically represents the inverse of the prompt reactor period. The mathematical properties of the resulting (linear) eigenvalue equation and the numerical schemes for assessing the dominant prompt α eigenvalue have been the subject of considerable research efforts: see for instance the comprehensive works (Bell and Glasstone, 1970; Duderstadt and Martin, 1979; Larsen and Zweifel, 1974; Albertoni and Montagnini, 1966) about theoretical aspects and (Hill, 1983; Brockway et al., 1985; Cullen et al., 2003; Singh et al., 2009; Yamamoto, 2011; Nolen et al., 2012; Zoia et al., 2014) concerning numerical methods.

For observation times t comparable to the decay lifetimes $\lambda_{i,j}^{-1}$ of the precursors, delayed contributions must be taken into account (Bell and Glasstone, 1970). It is customary to formally solve Eq. (8) for the precursor concentration and to replace the resulting $c_{\alpha}^{i,j}$ into Eq. (7). This yields the (nonlinear) eigenvalue problem for φ_{α} (Bell and Glasstone, 1970; Weinberg, 1952; Cohen, 1958; Henry, 1964)

$$\frac{\alpha}{\mathrm{v}}\varphi_{\alpha}(\mathbf{r},\mathbf{v})+L\varphi_{\alpha}(\mathbf{r},\mathbf{v})=F_p\varphi_{\alpha}(\mathbf{r},\mathbf{v})+\sum_{i,j}\frac{\lambda_{i,j}}{\lambda_{i,j}+\alpha}F_d^{i,j}\varphi_{\alpha}(\mathbf{r},\mathbf{v}), \tag{10}$$

where we have defined the delayed fission operator

$$F_d^{i,j}f=\chi_d^{i,j}(\mathbf{r},\mathrm{v})\int \nu_d^{i,j}(\mathrm{v}')\Sigma_f^{i}(\mathbf{r},\mathrm{v}')f(\mathbf{r},\mathbf{v}')\,\mathrm{d}\mathbf{v}'. \tag{11}$$

The full Eq. (10) including delayed contributions (in which case α is called the delayed time eigenvalue, and physically represents the inverse of the reactor period) has received comparatively less attention than the prompt Eq. (9) (see for instance (Bell and Glasstone, 1970; Cohen, 1958; Henry, 1964; Kaper, 1967) for a survey). However, Eq. (10) has recently attracted renewed interest (Hoogenboom, 2002; Betzler et al., 2012; Singh et al., 2011; Nauchi, 2014) in view of its practical applications in reactor kinetics. Indeed, integrating Eq. (10) over all phase space variables leads to the inhour-like equation

$$\rho_0=\alpha\Lambda_0+\sum_{i,j}\frac{\alpha}{\lambda_{i,j}+\alpha}\beta_0^{i,j} \tag{12}$$

where $\rho_0=(k_0-1)/k_0$ plays the role of a reactivity, with

$$k_0=\frac{\left\langle 1,F_p\varphi_{\alpha}+\sum_{i,j}F_d^{i,j}\varphi_{\alpha}\right\rangle}{\langle 1,L\varphi_{\alpha}\rangle}, \tag{13}$$

Λ_0 is the mean generation time

$$\Lambda_0=\frac{\left\langle 1,\frac{1}{\mathrm{v}}\varphi_{\alpha}\right\rangle}{\left\langle 1,F_p\varphi_{\alpha}+\sum_{i,j}F_d^{i,j}\varphi_{\alpha}\right\rangle}, \tag{14}$$

and $\beta_0^{i,j}$ are the flux-averaged delayed neutron fractions

$$\beta_0^{i,j}=\frac{\left\langle 1,F_d^{i,j}\varphi_{\alpha}\right\rangle}{\left\langle 1,F_p\varphi_{\alpha}+\sum_{i,j}F_d^{i,j}\varphi_{\alpha}\right\rangle}, \tag{15}$$

with $\sum_{i,j}\beta_0^{i,j}=\beta_0$. From Eq. (12) it is apparent that α represents the inverse reactor period in the inhour-like equation. The index zero is used to express the fact that these quantities have been averaged over the phase space variables without weighting by the adjoint flux.

In a recent paper, we have proposed a Monte Carlo method to find the dominant eigenvalue of Eq. (9), in the absence of delayed neutrons (Zoia et al., 2014). In this work, we extend our previous results to the general case of the non-linear eigenvalue problem of Eq. (10), including both prompt and delayed contributions. The paper is structured as follows: in Section 2 we first recall the algorithm for prompt eigenvalues and we show that it can be easily modified so as to include delayed neutrons. Then, in Section 3 we test the proposed algorithm on a few significant verification tests. A validation of the Monte Carlo calculations against reactor period measurements performed at the ORPHEE facility of CEA/Saclay is discussed in Section 4, and conclusions are finally drawn in Section 5.

2. A Monte Carlo method for alpha eigenvalues

The key tool for solving Eqs. (9) and (10) by Monte Carlo simulation is the so-called α-k power iteration algorithm,[1] whose specific details depend on the sign of the dominant eigenvalue α (Hill, 1983; Brockway et al., 1985; Cullen et al., 2003; Zoia et al., 2014). In the following, we sketch the structure of the algorithm by considering supercritical ($\alpha>0$) and subcritical ($\alpha<0$) configurations separately.

[1] Actually, alternative algorithms have been also proposed, such as the weight correction methods for prompt (Yamamoto, 2011) and delayed (Nauchi, 2014) α eigenvalues, or the transition rates matrix (Betzler et al., 2012).