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Calculation of half-value thickness for aluminum absorbers by means of fractional calculus

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ABSTRACT

Half-value thickness of aluminum absorbers has been investigated experimentally and theoretically. Cs-137, Tl-204 and Sr-90/Y-90 radio-isotopes were used as beta sources. Inconsistency between experimental measurements and standard theoretical calculations has been removed with the help of fractional calculus. The experimental and theoretical half-thickness values have been found equivalent for fractional derivative order ≈0.3.

1. Introduction

Transitions of electrons and positrons through different materials have been the subject of many studies since discovery of radioactivity (Leonard and Pobereskin, 1948; Seliger, 1952, 1955; Libby, 1956; Takhar, 1967, 1968; Thontadarya and Umakantha, 1971). When the beta particles pass through a material, such as aluminum, some of them are absorbed. Absorption rates depend on both the energy of the particles and thickness of material. The relationship between beta-ray intensity and thickness of absorber material is given by (Leo, 1994).

$$\frac{dI(x)}{dx} = -\mu_m I(x) \tag{1}$$

where I, μ_m (cm^2/g) and x (g/cm^2) are the intensity of the beta particle, the mass attenuation coefficient and the thickness of the absorber, respectively. μ_m is defined as the linear attenuation coefficient (μ), divided by the density of material (ρ):

$$\mu_m = \frac{\mu}{\rho}.$$

The standard solution of the above equation is

$$I(x) = I_0 \exp(-\mu_m x) \tag{2}$$

where I_0 represents initial intensity. Half-value thickness of an absorber is thickness of the absorber material where the intensity of beta particle entering it is reduced by one half and can be calculated as,

$$x_{1/2} = \frac{ln2}{\mu_m}. \tag{3}$$

Examination of the absorption of beta particles in different materials and analysis of mass attenuation coefficient of absorbers continues today (Batra and Sehgal, 1981; Ram et al., 1982; Burek and Chocyk, 1996; Gurler and Yalcin, 2005; La Rocca and Riggi, 2009; Ermis and Celiktas, 2012). A theoretical method for determining the range of electrons and positrons in Al, Cu and Au were reported by Batra and Sehgal (1981). A semi-empirical relation between maximum energy of electron and mass attenuation coefficient was derived, and mass attenuation coefficients and range of beta particles in Be, Al, Cu, Ag and Pb was investigated by Ram et al. (1982). A theory of mass attenuation coefficient was proposed and related parameters were computed by Burek and Chocyk (1996). Gurler and Yalcin (2005) obtained the mass attenuation coefficients of beta particles for Al, Cu and Au using a practical theoretical method. Mass attenuation coefficients of Al, brass and cardboard using ^{90}Sr were investigated and compared to GEANT simulations by La Rocca and Riggi (2009). Beta attenuation coefficients were determined by means of timing method by Ermis and Celiktas (2012).

In the present work, the decreasing intensity of beta particles, after they pass through the varying thicknesses of aluminum absorbers, has been experimentally investigated using $^{90}Sr/^{90}Y$, ^{137}Cs and ^{204}Tl radioisotopes. Mass attenuation coefficient and half-value thickness of Al absorber are found from the experimental data. Furthermore, half-value thickness has been theoretically

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calculated using Eq. (3). However, there is an inconsistency between the experimental and standard theoretical results of half-value thickness. To resolve this inconsistency, Eq. (1) represents the changing intensity of radiation as related to the thickness of the absorber has been solved using fractional calculus. Thus, even if different beta sources are used, a single fractional derivative order is obtained for aluminum absorber.

2. Fractional calculus

Fractional calculus generalizes the derivative of a function to a non-integer order (Oldham and Spanier, 1974; Miller and Ross, 1993; Podlubny, 1999). Within the 20th century, the numerical applications of physical problems have shown that fractional calculus is an alternative mathematical tool (Carpinteri and Mainardi, 1997; Hilfer, 2000).

Fractional mathematics and its some applications have been expressed by Oldham and Spanier (1974); Sabatier et al. (2007) and, Gorenflo and Mainardi (2008). The most commonly used definitions of fractional derivatives are Riemann-Liouville (RL), Caputo and Grünwald-Letnikov's fractional derivatives.

Fractional RL integral operator is given as:

$$J^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}d\tau \tag{4}$$

where α is any positive real number and $\Gamma(\alpha)$ is the Gamma function.

On the other hand, the RL fractional derivative operator is defined as,

$$D^{\alpha}f(t) := \begin{cases} \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau\right], & m-1<\alpha<m, \\ \frac{d^m}{dt^m}f(t), & \alpha = m. \end{cases} \tag{5}$$

The Caputo fractional derivative operator is given by the definition:

$$D_C^{\alpha}f(t) := \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau, & m-1<\alpha<m, \\ \frac{d^m}{dt^m}f(t), & \alpha = m \end{cases} \tag{6}$$

where α is the order of the fractional derivative and m is the smallest integer greater than α. The Caputo and the RL fractional derivative operators are not equivalent to each other;

$$D_C^{\alpha}f(t) = D^{\alpha}(f(t) - \sum_{k=0}^{m-1} f^{(k)}(0^+)). \tag{7}$$

Since the Caputo fractional derivative includes the initial values of function and its integer order derivatives (Podlubny, 1999), it is preferred to the RL derivative in physical applications. Though, the RL fractional derivative of a constant is different zero, in the Caputo fractional derivative it is zero (Podlubny, 1999; Carpinteri and Mainardi, 1997; Naber, 2004).

3. Fractional solution of attenuation equation

In order to establish the fractional attenuation equation, the first order time derivative is replaced by fractional derivative of order α in Eq. (1). Thus, the fractional attenuation equation is given by

$$D_C^{\alpha}I(x) = -\mu_m^{\alpha}I(x) \tag{8}$$

where D_C^{α} denotes the Caputo fractional derivative of order α. μ_m must be raised to the same order as the fractional derivative to preserve the dimension of Eq. (8) (Naber, 2004). For $\alpha = 1$, the fractional attenuation equation given by Eq. (8) reduces to the standard one given in Eq. (1).

To solve Eq. (8), the definitions in Eq. (7) is firstly used,

$$I(x) - \sum_{k=0}^{m-1} I^{(k)}(x)|_{x=0}\frac{x^k}{k!} = -\mu_m^{\alpha}J^{\alpha}I(x) \tag{9}$$

where $\sum_{k=0}^{m-1} I^{(k)}(x)|_{x=0} = I(0) = I_0$ represents the initial intensity. Eq. (9) can be re-written as

$$I(x) - I_0 = -\mu_m^{\alpha}J^{\alpha}I(x). \tag{10}$$

Then, the Laplace transform is performed on Eq. (10)

$$\tilde{I}(s) = I_0\frac{s^{\alpha-1}}{s^{\alpha}+\mu_m^{\alpha}} \tag{11}$$

where $\tilde{I}(s)$ is the Laplace transform of $I(x)$, and s is the Laplace transform parameter. One can express Eq. (11) as a series expansion

$$\tilde{I}(s) = I_0\sum_{k=0}^{\infty}\frac{(-1)^k\mu_m^{\alpha k}}{s^{\alpha k+1}}. \tag{12}$$

Performing the inverse Laplace transform to each of the terms of the series expansion in Eq. (12) following result is obtained

$$I(x) = I_0\sum_{k=0}^{\infty}\frac{(-1)^k\mu_m^{\alpha k}x^{\alpha k}}{\Gamma(\alpha k+1)} \tag{13}$$

$$I(x) = I_0E_{\alpha}(-\mu_m^{\alpha}x^{\alpha}) \tag{14}$$

where $E_{\alpha}(-\mu_m^{\alpha}x^{\alpha})$ represents the Mittag-Leffler function widely used in fractional calculations. This function is defined as,

$$E_{\alpha}(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(n\alpha+1)} \tag{15}$$

and $E_1(z) = \exp(z)$.

4. Experimental

The experiments were performed in Nuclear Research Laboratory of Physics Department of Dumlupınar University. The experimental setup includes a Geiger-Muller (GM) probe with sample holder and counter. The radioactive sources are $^{90}Sr/^{90}Y$, ^{137}Cs and ^{204}Tl beta sources, whose activity were $0.1\mu Ci$, $1\mu Ci$ and $1\mu Ci$, respectively. The dimensions of Al absorbers were $7\ \text{cm} \times 7\ \text{cm}$ and thickness of one absorber was $2.76\ \text{mg/cm}^2$. The distance between the absorber and the source was 1 cm. To conduct measurements, the absorbers were placed between beta sources and GM. Background and dead-time corrections were performed to obtain net beta counts.

5. Results and discussions

The experimental and calculated values (i.e. standard solution of Eq. (1)) of half-value thicknesses of Al absorber for different beta particle sources were not equivalent. To resolve this discrepency, Eq. (1) has been defined using Caputo fractional derivative as Eq. (8) and the solution of Eq. (8) has been obtained as Eq. (14).

The experimental measurements of change in intensity of beta particle versus the thickness of absorbers are shown for $^{90}Sr/^{90}Y$, ^{137}Cs and ^{204}Tl beta sources in Fig. 1. As can be seen in the figure, the initial intensity, its half-value and half-value thickness of Al absorber are $I_0 = 6268$, $\frac{I_0}{2} = 3134$, $x_{1/2} = 157.1500\ \text{mg/cm}^2$ for $^{90}Sr/^{90}Y$, $I_0 = 29830$, $\frac{I_0}{2} = 14915$, $x_{1/2} = 44.4296\ \text{mg/cm}^2$ for ^{137}Cs and $I_0 = 2182$, $\frac{I_0}{2} = 1091$, $x_{1/2} = 34.6977\ \text{mg/cm}^2$ for ^{204}Tl, respectively. In Fig. 2, logarithm of the data in Fig. 1 is shown. The mass attenuation coefficients of the absorbers for different sources have been