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Interpretations of alternative uncertainty representations in a reliability and risk analysis context

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ABSTRACT

Probability is the predominant tool used to measure uncertainties in reliability and risk analyses. However, other representations also exist, including imprecise (interval) probability, fuzzy probability and representations based on the theories of evidence (belief functions) and possibility. Many researchers in the field are strong proponents of these alternative methods, but some are also sceptical. In this paper, we address one basic requirement set for quantitative measures of uncertainty: the interpretation needed to explain what an uncertainty number expresses. We question to what extent the various measures meet this requirement. Comparisons are made with probabilistic analysis, where uncertainty is represented by subjective probabilities, using either a betting interpretation or a reference to an uncertainty standard interpretation. By distinguishing between chances (expressing variation) and subjective probabilities, new insights are gained into the link between the alternative uncertainty representations and probability.

1. Introduction

The setting analysed in this paper is the standard set-up for an uncertainty analysis [1,2]: A quantity of interest Z has been identified. To assess Z, a model $G(X)$ is introduced which links a set of input variables X to Z. To describe the uncertainties U about X and Z, probabilistic and non-probabilistic methods can be used. Propagating uncertainty about X through the model G, an uncertainty description is obtained for Z. The tool used for this purpose could be an analytical approach or Monte Carlo simulation. Measures such as expected values and quantiles are computed from the uncertainty distributions produced, and these measures provide an input to a decision process, which could be based on some decision criteria expressing, for example, that a probability should not exceed a specified level. Sensitivity analysis provides insights about how the input quantities affect the output quantities, and importance ranking identifies what factors, subsystems, etc. are most important based on some defined criteria, for example, the contribution to the expected value of Z. The result of the analysis may lead to some action (feedback process), for example, that there is a need for design changes to meet the criteria.

Our main interest is reliability and risk applications, as presented for example in [3–9], but the setting studied is general and extends beyond these applications. For reliability and risk applications, Z and X could, for example, represent costs, the number of fatalities, the occurrence of a system failure, the fraction of failed units in a large population of similar units, or the distribution of failure times in a large population of similar units. The two last examples are referred to as a chance and chance distributions in a Bayesian context [10]. In a traditional statistical framework, they are relative frequency-interpreted probabilities. The quantities are unknown and in the uncertainty (risk, reliability) analysis, the uncertainties are assessed.

This points to the common distinction made in reliability and risk analysis between aleatory uncertainties (represented by the probability models and chances) and the epistemic uncertainties expressing lack of knowledge about the "true" value of the chances and parameters of the probability models [11–17].

Probability is the common tool used for representing the epistemic uncertainties about X and Z, but a probability has different interpretations. One such interpretation is to consider this probability as a subjective probability with reference to a standard expressing the analysts' uncertainty about X and Z. Following this interpretation, the assessor compares his/her uncertainty about the occurrence of the event A with the standard event of drawing at random a favourable ball from an urn that contains $P(A)\times 100\%$ favourable balls [18]. A subjective probability is based on some knowledge (K) (often referred to as the background knowledge), and to highlight this dependency, we write $P(A)=P(A|K)$. The background knowledge comprises assumptions and suppositions, models, etc.

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However, a subjective probability can also be given other interpretations. Among economists and decision analysts, and the earlier probability theorists, a subjective probability is linked to betting. According to this perspective, the probability of the event A, $P(A)$, equals the amount of money that the assigner would be willing to put on the table if he/she would receive a single unit of payment in the case that the event A was to occur, and nothing otherwise. The opposite must also hold, i.e. the assessor must be willing to pay the amount $1-P(A)$ if he/she would receive a single unit of payment in the case that A was not to occur, and nothing otherwise. In other words, the probability of an event is the price at which the person assigning the probability is neutral between buying and selling a ticket that is worth one unit of payment if the event occurs, and worthless if not [10]. For some related interpretations of subjective probabilities, see [10,19–21].

Thus meaningful interpretations of a subjective probability exist. What type of interpretation that should be used is a debated topic, see e.g. [22–23]. According to these references, only the reference to the uncertainty standard interpretation should be used: the betting interpretation is not considered appropriate to describe uncertainties as it extends beyond the realm of uncertainty assessments—it reflects the assessor's attitude to money and the gambling situation, which means that analysis (evidence) is mixed with values. In this view, the scientific basis for uncertainty (risk, reliability) assessment is founded on the idea that professional analysts describe uncertainty (risk, reliability) separated from how we (the assessor, the decision-maker or other stakeholders) value the consequences and the uncertainty (risk, reliability). However, other researcher and analysts prefer the betting interpretation or related interpretations. Their focus is on decision-making under uncertainty, and then the betting situation applies: the distinction between uncertainty assessment and value judgments is not important. In the following, when we refer to a subjective probability, either the reference to the uncertainty standard interpretation or the betting interpretation is applied.

Many other representations of uncertainty exist, including imprecise (interval) probability, fuzzy probability and representations based on the theories of evidence (belief functions) and possibility. In recent years, such representations have been given considerable attention among researchers and analysts, see e.g. [1,24,25]. It is argued that these representations are more satisfactory in describing imprecise information than probability. Others argue however that probability theory provides the appropriate mathematical structure for the representation of uncertainty and that no other is needed [18,26]. A typical statement is [27]: "for me, the introduction of alternatives such as an interval analysis to standard probability theory seems a step in the wrong direction, and I am not yet persuaded it is a useful area even for the theoretical research. I believe risk analysts will be better off using standard probability theory than trying out alternatives that are harder to understand, and which will not be logically consistent if they are not equivalent to standard probability theory".

In this paper, we do not take a stand what is the best representation of uncertainty. We simply accept that there exists a number of ways uncertainty can be represented, and we would like to clarify to what extent these meet one basic requirement set for such representations: the interpretation needed to explain what an uncertainty number expresses.

Three requirements are normally put forward to such representations ([3, p. 20]):

i. *Axioms:* specifying the formal properties of the uncertainty representation.
ii. *Interpretations*: connecting the primitive terms in the axioms with observable phenomena.
iii. *Measurement procedures*: providing, together with supplementary assumptions, practical methods for interpreting the axiom system.

Most uncertainty representations meet the first criterion, but many struggle with the criteria (ii) and (iii) and in particular the interpretation criterion (ii). We should not use a representation, which has no clear interpretation. It is not sufficient to say that a measure expresses a degree of something. We need to know what it means that the measure is 0.2 instead of 0.4. If such an interpretation cannot be provided, the result is a number crunching exercise, which cannot be used in a scientific analysis. For the subjective probability, we can give a precise interpretation as indicated above: $P(A)=0.1$ of an event A means that the assessor compares the uncertainty (degree of belief) with drawing at random one particular ball out of an urn, which comprises 10 balls. This means that the assessor considers the uncertainties about the following two events equivalent: the occurrence of A, and drawing a favourable ball from an urn containing 10% favourable balls. Alternatively, a betting interpretation can be given.

The present paper is motivated by Cooke [28] who asks for operational definitions of the degree of possibility and related notions. We seek to bring the discussion one step further by looking in detail into some of the prevailing interpretations. Our main focus is on the use of various types of intervals to bound probabilities. What do these bounds mean in the context defined above? How do these bounds relate to the subjective probabilities (interpreted either with reference to an uncertainty standard or through betting)? In the literature on alternative uncertainty representations, probability is typically viewed as a relative frequency-interpreted probability or a subjective probability with a betting interpretation [1,24,28,29]. Baudrit and Dubois [29] write:

> …Bayesian subjectivist approach maintains that only a standard probabilistic representation of uncertainty is rational, but this claim relies on a betting interpretation that enforces the use of a single probability distribution, in the scope of decision-making, not with a view to faithfully report the epistemic state of an agent.

In the paper, we specifically address how the alternative representations are to be interpreted, when the reference to an uncertainty standard is applied.

An interesting issue is to what extent the alternative uncertainty representations provide a more faithful report of the uncertainties than using subjective probabilities and in particular credibility intervals following Bayesian paradigm. A 90% (say) credibility interval for a parameter θ is an interval $[a,b]$ such that θ is in the interval with probability 0.90 (according to the subjective probability).

A detailed analysis of this issue to what extent the alternative uncertainty representations provide a more faithful report of the uncertainties than using subjective probabilities is however outside the scope of the present paper, reference is made to discussions in [23,30,31]. The main aim of the present paper is to clarify and discuss the interpretation of the various uncertainty representations. The aim is to provide an improved basis for making judgements about the appropriateness of the various measures.

One of the reviewers of the present paper suggested adding a discussion on how uncertainty representations should be interpreted when they are obtained by aggregating multiple individual uncertainty representations (e.g. uncertainty representations provided by multiple independent experts). This is an interesting topic, but is also beyond the scope of the present paper. The topic is related to whose uncertainty assessments a reliability or risk analysis reports: the analysts' or the experts'? In the present paper,