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Sufficient conditions for globally asymptotic self-stability of pressurized water reactors

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ABSTRACT

After the Fukushima accident, safe, stable and efficient operation of reactors is very necessary for the development of nuclear power industry. Since pressurized water reactor (PWR) is the mostly widely used fission reactor, the improvement of its operation performance is quite meaningful. Self-stability is the most important dynamic feature of any reactors, and analyzing the self-stability can give the approach of improving the operation performance. With this in mind, the self-stability analysis of the PWR is presented through the shifted-ectropy based approach, and sufficient conditions for the globally asymptotic self-stability in cases of negative, zero and positive coolant temperature feedback coefficient are all established. The correctness of the theoretical results are finally verified through numerical simulation. The results of this paper give the way to not only guaranteeing self-stability through physical and thermal-hydraulic reactor design but also strengthening closed-loop stability and robustness by the means of feedback control.

1. Introduction

Stimulated by the growing electricity requirement and the resulting pollution of burning the fossil fuels, developing clean and green energy technology has already been seen as the basis for the sustainable development of human society. With comparison to those new energies such as wind energy, and solar energy, nuclear fission energy is still the sole one that can substitute the fossil in a centralized way and in a great amount with commercial availability and economic competitiveness, and will certainly play a more prominent role in the world energy supply. Continued and expanded reliance on nuclear energy gives the current rebirth of nuclear industry for electricity production which strongly depends on safe and effective control strategies. After the Fukushima nuclear accident, safe and stable operation of nuclear reactors has become more important than before. Power-level regulation is one of those techniques that guarantee safe, stable and efficient operation. Since the PWR is the most widely used nuclear power reactors, its power-level regulation is undoubtedly meaningful. In the past decades, due to the development of advanced digital control platforms, some promising advanced power-level regulators were given for PWRs. Edwards et al. (1990) presented a state feedback assisted classical control (SFAC) strategy for power-level control of nuclear reactors. The key feature of the SFAC is using a well-designed regulation strategy composed of a state feedback controller and a state estimator to modify the load signal for an embedded classical output feedback power-level controller, which results in easy implementation. After establishing the SFAC, some further works were done for improving the control performance of this control strategy (Ben-Abdennour et al., 1992; Arab-Alibeik and Setayeshi, 2003). However, the current version of the SFAC is based on classical linear control theory. Since the nuclear reactors are actually complex nonlinear dynamic systems, there still exists developing space for the SFAC. For compensating the nonlinear dynamics of the PWR, artificial neural networks are utilized to design nonlinear power-level control strategies (Ku et al., 1992; Arab-Alibeik and Setayeshi, 2005). Moreover, since the model predictive control (MPC) technique is a powerful tool for regulating industrial processes, it has also been applied to the power-level control of the PWRs (Na et al., 2006; Eliasi et al., 2012). Based upon the backstepping technique (Kokotović, 1992; Krstić et al., 1995) and the dissipation-based high gain filter (DHGF) (Dong et al., 2010), a nonlinear dynamic output-feedback power-level control was given for the PWRs (Dong, 2011; Dong et al., 2011). These power control strategies guarantee or strengthen the closed-loop stability of the PWR plants through feedback. Now, the question is that is the PWR itself stable? Since the self-stability is the most important dynamic features of the PWR, it is so necessary to give the self-stability analysis, from which we can know the path to strengthen the closed-loop stability.

Study on the self-stability of nuclear reactors began in 1950s, and is still a hot topic in nuclear engineering until now. Based on building the mechanical analogy and defining the Hamiltonian functions for nuclear reactor dynamics (Ergen and Weinberg, 1954), the reactor self-stability is analyzed by the use of Lyapunov

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second method (Ergen et al., 1957). However, the effect of delayed neutrons was neglected in this work. Topological method was also applied to give the self-stability analysis for those homogeneous reactors (Smets and Gyftopoulos, 1959). Devooght and Smets (1967) applied the topological method, the Lyapunov method, the Aizermann's method and the Rosen method to the self-stability for homogeneous reactors, and compared the results giving by these approaches. As discussed above, the renaissance of nuclear energy relies on safe, stable and efficient operation of nuclear power plants, which recover the research on the self-stability of nuclear reactors. Considering the delayed neutron kinetics and temperature feedback, Theler and Bonetto (2010) gave the linear stability analysis for point kinetics reactors. Based on the concept of shifted-ectropy and Lyapunov second method, Dong (2012) proved theoretically that high temperature gas-cooled reactor (HTGR) has the crucial feature of globally asymptotic self-stability (GASS). Therefore, it is meaningful to give the condition of GASS for the PWR. However, since there are big differences between that dynamics of the PWR and that of the HTGR. For HTGR, the reactivity feedback effect is caused by the temperatures of both the fuel elements and the reflector. With comparison to the former one, the latter one is weaker, and its time constant is much larger, which means that the reactivity feedback effect of the reflector temperature can be omitted in the self-stability of the HTGR. However, for the PWR-like reactors, the temperature feedback effect of the coolant is usually stronger than that of the fuel elements (Oka and Suzuki, 2013), and the time constant difference between these two temperature feedback effects of the PWR is much smaller than that between the temperature feedback effects corresponding to the fuel elements and the reflector of the HTGR. Thus, the reactivity feedback effects caused by both the fuel and the coolant temperatures must be considered simultaneously in the self-stability analysis, and the method for analyzing HTGR self-stability cannot be applied directly to those PWR-like reactors.

In this paper, based upon the concept of shifted-ectropy, the Lyapunov function for analyzing the PWR self-stability is given, and then sufficient conditions of GASS are established. Finally, numerical simulation results verify the correctness of the theoretical results. Here, both the effects caused by the delayed neutron and nonlinearity induced by temperature feedback are considered in this work.

2. Modeling and problem formulation

2.1. Dynamic model for self-stability analysis

The reactor dynamics adopted here is the point kinetics with one equivalent delayed neutron group and the reactivity feedback given by the variation of average temperatures of the fuel and coolant (Schultz, 1961; Edwards et al., 1990; Dong et al., 2009), which is given as follows:

$$\begin{cases} \dot{n}_\mathrm{r} = \frac{1}{\Lambda}[(\rho_\mathrm{r}-\beta)n_\mathrm{r}+\beta c_\mathrm{r}+\alpha_\mathrm{f} n_\mathrm{r}(T_\mathrm{f}-T_\mathrm{f,m})+\alpha_\mathrm{c} n_\mathrm{r}(T_\mathrm{cav}-T_\mathrm{cav,m})], \\ \dot{c}_\mathrm{r} = \lambda(n_\mathrm{r}-c_\mathrm{r}), \\ \dot{T}_\mathrm{f} = \frac{1}{\mu_\mathrm{f}}[-\Omega(T_\mathrm{f}-T_\mathrm{cav})+P_0 n_\mathrm{r}], \\ \dot{T}_\mathrm{cav} = \frac{1}{\mu_\mathrm{c}}[\Omega(T_\mathrm{f}-T_\mathrm{cav})-2M(T_\mathrm{cav}-T_\mathrm{cin})]. \end{cases} \tag{1}$$

where n_r is the neutron density relative to density at rated condition, c_r is the density of the delayed neutron precursor relative to the density at rated condition, β is the fraction of delayed fission neutrons, Λ is the effective prompt neutron lifetime, λ is the effective radioactive decay constant of the delayed neutron precursor, α_f and α_c are respectively the reactivity coefficients of the fuel and coolant temperatures, T_f is the average fuel temperature, T_cav and T_cin are respectively the average temperature of the coolant inside the core and the temperature of the coolant entering the core, $T_\mathrm{cav,m}$ and $T_\mathrm{f,m}$ are respectively the initial equilibrium values of T_cav and T_f, Ω is the heat transfer coefficient between fuel and coolant, M is the mass flow rate times heat capacity of the coolant, P_0 is the rated power level, ρ_r is the reactivity due to the control rods, μ_f is the total heat capacity of the fuel, μ_c is the total heat capacity of the reactor coolant.

The deviations of the actual values of n_r, c_r, T_f, T_cav, T_cin and ρ_r from their steady values, i.e. n_r0, c_r0, T_f0, T_cav0, T_cin0 and ρ_r0 are concerned, which are defined respectively as

$$\begin{cases} \delta n_\mathrm{r} = n_\mathrm{r} - n_\mathrm{r0}, \\ \delta c_\mathrm{r} = c_\mathrm{r} - c_\mathrm{r0}, \\ \delta T_\mathrm{f} = T_\mathrm{f} - T_\mathrm{f0}, \\ \delta T_\mathrm{cav} = T_\mathrm{cav} - T_\mathrm{cav0}, \\ \delta \rho_\mathrm{r} = \rho_\mathrm{r} - \rho_\mathrm{r0}, \\ \delta T_\mathrm{cin} = T_\mathrm{cin} - T_\mathrm{cin0}. \end{cases} \tag{2}$$

For analyzing the stability of the steady point, it is not loss of generality to assume that

$$\delta\rho_\mathrm{r} \equiv 0. \tag{3}$$

Moreover, we also omit the influence of the secondary loop to the primary loop, i.e. assume that

$$\delta T_\mathrm{cin} \equiv 0. \tag{4}$$

Moreover, define

$$\boldsymbol{x} = [\,\delta n_\mathrm{r} \quad \delta c_\mathrm{r} \quad \delta T_\mathrm{f} \quad \delta T_\mathrm{cav}\,]^\mathrm{T}, \tag{5}$$

and from (1) and (2) and assumptions (3) and (4), the dynamic model for self-stability analysis can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \tag{6}$$

where

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} -\frac{\beta}{\Lambda}(x_1-x_2)+\frac{n_\mathrm{r0}+x_1}{\Lambda}(\alpha_\mathrm{f}x_3+\alpha_\mathrm{c}x_4) \\ \lambda(x_1-x_2) \\ -\frac{\Omega}{\mu_\mathrm{f}}(x_3-x_4)+\frac{P_0}{\mu_\mathrm{f}}x_1 \\ \frac{\Omega}{\mu_\mathrm{c}}(x_3-x_4)-\frac{2M}{\mu_\mathrm{c}}x_4 \end{bmatrix}. \tag{7}$$

2.2. Problem formulation

Furthermore, it is easy to see that analyzing the stability of steady points is equivalent to analyzing the stability of the origin of autonomous nonlinear system (6), and the theoretical problem to be solved in this paper is given as follows:

Problem 1. Under what condition is the origin of system (6) globally asymptotically stable?

3. Shifted-ectropy of thermodynamic systems

Since a PWR is essentially a thermodynamic system with nuclear fission reaction, it is necessary to study the self-stability of general thermodynamic systems firstly. The result of this section is a promotion of that in Dong (2012), and here we only introduce these results briefly.

Consider thermodynamic system G which involves energy exchange between q interconnected subsystems. The structure of G is shown in Fig. 1. Let E_i be a nonnegative quantity denoting the energy of the ith subsystem, S_i: $[0,\infty) \to R$ be the power supplied to (or extracted from) the ith subsystem, σ_{ij}: $[0,\infty) \to [0,\infty)$, $i \neq j$, i, $j = 1, \ldots, q$, be the instantaneous rate of energy flow from the jth to the ith subsystem, and σ_{ii}: $[0,\infty) \to [0,\infty)$, $i = 1, \ldots, q$, be the