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An alternative stochastic formulation for the point reactor

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ABSTRACT

The stochastic behavior of a point reactor is modeled with a system of Ito stochastic differential equations. This new approach does not require computing the square root of a matrix which is a great computational advantage. Moreover, the derivation procedure clearly demonstrates the mathematical approximations involved in the final formulation. Three numerical benchmarks show the accuracy of this model in predicting the mean and variance of the neutron and precursor population in a point reactor.

1. Introduction

It is well known that the reactions in a nuclear system are not fully describable by deterministic laws. This fact, at the most fundamental level, is due to the laws of quantum mechanics, which only give probabilities of various interactions for a neutron, which are manifest in the interaction cross sections of atoms with neutrons. There are various situations which this probabilistic behavior could be readily observed for a nuclear system, e.g. in the startup of the reactors, in zero power reactors, in most laboratory source – detector configurations, etc.

There has been an extensive research effort to model this stochastic behavior. Some researchers have attempted to give a description based on the quantum mechanical laws of evolution in such a system (Osborn, 1969; Osborn and Yip, 1966). Others have tried to obtain suitable master equations for the system under study, which embed the stochastic information of the system (Degweker, 1994; Lewins, 1978; Pázsit and Pál, 2008). Some have tried to attribute this stochastic behavior to an (imaginary) fluctuation source, and have developed the so-called Langevinian description (Akcasu and Stolle, 1989; Gençay and Akcasu, 1981; Williams, 1974). These models generally trade simplicity for accuracy.

However, the most accurate formulation of the stochastic processes involved may not always be useful. Measuring higher order moments requires more data from the system for a given accuracy. Actually, most of the times in practice, one only measures the first and second order moments in a system, i.e. the mean and variance. The Central Limit Theorem (CLT) assures the accuracy of such an examination from a mathematical perspective (Eagleson, 1975). This fact has manifest itself in recent stochastic research trends, namely the Stochastic Differential Equation (SDE) formulation (Allen, 2007; Ikeda and Watanabe, 1989; Øksendal, 2003). Formulations based on this method are capable of describing stochastic processes up to the second moment, which is enough in most applications.

Most of these formulations have two main disadvantages. First, they may not be quite satisfactory from a physicist's perspective, in the rather sudden appearance of Weiner processes. Second, the need to compute the square root of the covariance matrix in these formulations makes these undesirable from a computational point of view. This research describes an alternative SDE developed for a point reactor system, namely the Simplified Stochastic Point Kinetics equation (SSPK), based on (Gillespie, 2000) methodology.

The current research is organized in six sections. In the next section, the stochastic processes involved in a point reactor are described. In the third section, the methodology for obtaining an SDE, assuming that the reactions of the system are known, is described. The fourth section deals with applying this methodology to a point reactor system with multiple precursor groups. In the fifth section, numerical benchmarks are given which show the accuracy of this method. Finally, the results are discussed and further improvements are suggested.

2. The stochastic point reactor

A point reactor is a reactor in which the spatial effects have been eliminated. This is obviously possible if the reactors length is infinite in all spatial dimensions. However definitions exist

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which eliminate the need for such an (unphysical) requirement (Henry, 1975). Fortunately, the final form in all of these formulations is alike. Namely, for a reactor with J precursor groups, one may write

$$\frac{d}{dt}\begin{bmatrix}\bar{n}\\ \bar{c}_1\\ \vdots\\ \bar{c}_J\end{bmatrix}=\begin{bmatrix}\frac{\rho-\sum_{j=1}^{J}\beta_j}{\Lambda} & \lambda_1 & \dots & \lambda_J\\ \frac{\beta_1}{\Lambda} & -\lambda_1 & & \\ \vdots & & & \\ \frac{\beta_J}{\Lambda} & 0 & & \end{bmatrix}\begin{bmatrix}\bar{n}\\ \bar{c}_1\\ \vdots\\ \bar{c}_J\end{bmatrix}+\begin{bmatrix}q\\ 0\\ \vdots\\ 0\end{bmatrix}, \tag{1}$$

where in Eq. (1), n and c_j are the neutron and jth precursor group populations, q is the strength of an external neutron source, ρ is the reactors reactivity, β_j and λ_j are the delayed neutron fission fraction and decay constant for the jth precursor group, Λ is the prompt neutron lifetime (Hetrick, 1993), and the bar indicates the ensemble averaging operator.

Study of a point reactor, i.e. studying the properties of Eq. (1), is desirable in the sense that it captures some of the most essential features of the reactor dynamics without involving into the complexities of integro-differential equations, i.e. the transport equation, or partial differential equations, i.e. the diffusion equation. However, as it is evident from Eq. (1), this formulation does not contain any stochastic terms, hence it could not describe the fluctuations in neutron and precursor populations. As a matter of fact, this equation should be interpreted as the ensemble averaged quantities in infinite alike reactors (Bell, 1969).

The starting point in accurately modeling the stochastic processes in a point reactor is the consideration of basic processes involved in a small time step δt. These processes are listed in Table 1.

In this table, v is the mean velocity of neutrons, Σ_c is the probability per unit length of neutron capture and $\Sigma_{f,i_1,i_2,\dots,i_{J+1}}$ is the probability per unit length of a neutron to induce a fission which yields i_1 prompt neutrons and i_{j+1} precursors for the jth precursor group.

Using the data embedded in Table 1, the stochastic point reactor could be readily simulated by the Monte Carlo (MC) method, which the results may then be used to extract the stochastic properties of such a reactor. Unfortunately in this method, one relies heavily on simulations and could gain little information a priori on the stochastic properties. In other words, to obtain the moments evolution, even the mean behavior, needs a full MC simulation.

A circumvent to these drawbacks is using an SDE model. Usually, in this method, one uses the model (Allen, 2007)

$$d\mathbf{x}=\mathbf{m}dt+C^{\frac{1}{2}}d\mathbf{w}, \tag{2}$$

where in Eq. (2), $\mathbf{w}$ is a $J+1$ dimensional Wiener process, $\mathbf{m}$ is the mean vector, and C is the covariance matrix, for reasons which are seen in the following. From the properties of Wiener processes,z namely

$$\begin{aligned}&E(d\mathbf{w})=0,\\ &E(d\mathbf{w}d\mathbf{w}^{\mathrm{T}})=Idt,\end{aligned} \tag{3}$$

and by noting the self-adjointness of the covariance matrix, it could be seen that

$$\begin{aligned}&E(d\mathbf{x})=\mathbf{m}dt,\\ &E(d\mathbf{x}d\mathbf{x}^{\mathrm{T}})=Cdt.\end{aligned} \tag{4}$$

Eq. (4) readily justifies the terms chosen for $\mathbf{m}$ and C. Using the data in Table 1, one could use Eq. (4) to readily compute $\mathbf{m}$ and C, as in (Allen, 2007; Hayes and Allen, 2005; Saha Ray, 2012). Such a computation leads to (Hayes and Allen, 2005)

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix}n\\ c_1\\ \vdots\\ c_J\end{bmatrix}&=\begin{bmatrix}\frac{\rho-\sum_{j=1}^{J}\beta_j}{\Lambda} & \lambda_1 & \dots & \lambda_J\\ \frac{\beta_1}{\Lambda} & -\lambda_1 & & \\ \vdots & & & \\ \frac{\beta_J}{\Lambda} & 0 & & \end{bmatrix}\begin{bmatrix}n\\ c_1\\ \vdots\\ c_J\end{bmatrix}+\begin{bmatrix}q\\ 0\\ \vdots\\ 0\end{bmatrix}\\ &+\begin{bmatrix}\zeta & a_1 & \dots & a_J\\ a_1 & r_1 & & \\ \vdots & & & \\ a_J & b_{J,2} & & \end{bmatrix}^{\frac{1}{2}}d\mathbf{w},\end{aligned} \tag{5}$$

where in Eq. (5)

$$\begin{aligned}\zeta&=\gamma n+\sum_{j=1}^{J}\lambda_j c_j+q,\\ \gamma&=\frac{-1-\rho+2\sum_{j=1}^{J}\beta_j+\left(1-\sum_{j=1}^{J}\beta_j\right)^2 v}{\Lambda},\\ a_i&=\frac{\beta_i\left(-1+\left(1-\sum_{j=1}^{J}\beta_j\right)v\right)}{\Lambda}n-\lambda_i c_i,\\ b_{i,j}&=\frac{\beta_{i-1}\beta_{j-1}v}{\Lambda}n,\\ r_i&=\frac{\beta_i^2 v}{\Lambda}n+\lambda_i c_i.\end{aligned} \tag{6}$$

While this model takes account of the stochastic process involved, using a Wiener process term, routine application of this model is difficult due to the need to compute the square root of the covariance matrix in each step. Note that this process is computationally expensive and may result in non-real values for the physical quantities due to numerical round offs. Also, the reason for introducing the Wiener processes in this model may not be readily clear and be a source for confusion. In the next section we aim at eliminating these drawbacks.

3. An alternative SDE modeling

Assume that the stochastic quantities in a system are shown by the vector $\mathbf{x}$. Assuming the system to follow a Markov model, for the ith component of such a system, one may write

$$x_i(t_0+\tau)=x_i(t_0)+\sum_{r\in\text{All reactions}}R_r(\mathbf{x}(t_0),\tau)v_{r,i}, \tag{7}$$

where in Eq. (7), we have represented the total number of r-type reactions starting from the state $\mathbf{x}(t_0)$ after a time τ by $R_r(\mathbf{x}(t_0),\tau)$. Also, the number of i-type particles generated from a r-type

Table 1
Basic processes in a point reactor in an infinitesimal time interval.

	Probability rate of occurrence	Change in neutron population	Change in jth precursor group population
Radiative capture	$n\nu\Sigma_c$	-1	0
Fission that results in i_1 neutrons and $i_2,\dots,i_{j+1}$ precursors	$n\nu\Sigma_{f,i_1,i_2,\dots,i_{j+1}}$	i_1-1	i_{j+1}
Decay for a precursor in group j	$c_j\lambda_j$	+1	-1
External source emission	q	+1	0