Annals of Nuclear Energy 63 (2014) 711–720

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Frequency domain Monte Carlo simulation method for cross power spectral density driven by periodically pulsed spallation neutron source using complex-valued weight Monte Carlo

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ARTICLE INFO

Article history:
Received 12 June 2013
Received in revised form 8 September 2013
Accepted 11 September 2013
Available online 10 October 2013

Keywords:
Monte Carlo
Neutron noise
Cross power spectral density
Frequency domain
ADS

ABSTRACT

In an accelerator driven system (ADS), pulsed spallation neutrons are injected at a constant frequency. The cross power spectral density (CPSD), which can be used for monitoring the subcriticality of the ADS, is composed of the correlated and uncorrelated components. The uncorrelated component is described by a series of the Dirac delta functions that occur at the integer multiples of the pulse repetition frequency. In the present paper, a Monte Carlo method to solve the Fourier transformed neutron transport equation with a periodically pulsed neutron source term has been developed to obtain the CPSD in ADSs. Since the Fourier transformed flux is a complex-valued quantity, the Monte Carlo method introduces complex-valued weights to solve the Fourier transformed equation. The Monte Carlo algorithm used in this paper is similar to the one that was developed by the author of this paper to calculate the neutron noise caused by cross section perturbations. The newly-developed Monte Carlo algorithm is benchmarked to the conventional time domain Monte Carlo simulation technique. The CPSDs are obtained both with the newly-developed frequency domain Monte Carlo method and the conventional time domain Monte Carlo method for a one-dimensional infinite slab. The CPSDs obtained with the frequency domain Monte Carlo method agree well with those with the time domain method. The higher order mode effects on the CPSD in an ADS with a periodically pulsed neutron source are discussed.

1. Introduction

Many theoretical and experimental studies on monitoring the subcriticality of accelerator-driven systems (ADS) using reactor noise techniques have been performed thus far. The features of the noise measurements in ADSs are that multiple neutrons are emitted from the spallation target at a single spallation event and that pulsed neutrons are emitted deterministically and periodically (i.e., non-Poisson). Due to these features, a classical reactor noise measurement theory cannot be directly applied to the subcritical measurements in ADSs. The theoretical formula for Feynman-α method or Rossi-α method in ADSs were studied by, for example, Pázsit et al. (2004), Kitamura et al. (2006), and Muñoz-Cobo et al. (2008). Another technique that uses the auto power spectral density (APSD) or cross power spectral density (CPSD) were studied by, for example, Muñoz-cobo et al. (2001), Rugama et al., (2002), Ballester and Muñoz-Cobo (2006), and Degweker and Rana (2007). Sakon et al. (2013) very recently carried out a series of power spectral analyses in a thermal subcritical reactor system driven by a periodic and pulsed 14 MeV neutron source at the Kyoto University Critical Assembly (KUCA). The CPSD driven by a periodically pulsed neutron source is composed of two components: a correlated component and an uncorrelated component of an infinite number of the Dirac delta function peaks (Ballester and Muñoz-Cobo (2006), Sakon et al. (2013)). The fundamental mode time-decay constant α of an ADS can be obtained by fitting either of two components to the conventional theoretical formula for the CPSD to the extent that the fundamental mode approximation yields reasonably acceptable results.

Monte Carlo simulation techniques have been developed for these reactor noise measurements. Some special functions that enable the simulation of reactor noise measurements have been added to existing production Monte Carlo codes (Ficaro and Wehe, 1994; Valentine and Mihalczo, 1996; Nagaya et al., 2005). In these Monte Carlo calculations, time series data that record neutron or gamma ray detections are stored during the course of the random walk processes for simulating the subcriticality measurements. Then, the time series data are processed to obtain the Feynman Y function or power spectral density. Continuous energy Monte Carlo codes such as MCNP and MVP can simulate complicated experimental configurations within a reasonable degree of accuracy. Thus, the Monte Carlo simulation can perform the faithful reproduction of subcriticality measurements, which contributes to the

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http://dx.doi.org/10.1016/j.anucene.2013.09.024

planning of experiments or to reviewing the experimental results. Besides, the Monte Carlo simulations can be used to validate the calculational methods and cross section data sets with measured data from subcritical experiments. Even with the latest high performance computers, however, a time-dependent Monte Carlo simulation for a subcriticality measurement sometimes requires lengthy computer time.

If one is interested in calculating the power spectral density with the Monte Carlo method, a time-dependent (i.e., time domain) Monte Carlo calculation is expected to be avoided by introducing a Monte Carlo method for frequency domain analyses. Neutron noise calculations in the frequency domain can be performed by solving a Fourier transformed neutron transport or diffusion equation. A technique for solving the Fourier transformed diffusion equation has long been used for diagnostic purposes in power reactor cores (Pázsit and Demazière, 2010; Demazière, 2011). On the other hand, a method for solving the Fourier transformed transport equation was first developed with the Monte Carlo method by Yamamoto (2013). A new Monte Carlo algorithm that uses complex-valued weights (Yamamoto, 2012) is introduced to solve the equation. While the neutron noise in (Yamamoto, 2013) is induced by cross section perturbations, the frequency domain neutron flux in an ADS, which is to be obtained from the Fourier transformed transport equation, is caused by the time variation of the external neutron source.

The author of the present paper tries to apply the complex-valued weight Monte Carlo method to obtaining the Fourier transformed neutron flux in an ADS. CPSDs obtained with the newly-developed Monte Carlo method are to be compared with those with time-dependent (time domain) Monte Carlo calculations for verification of the new method.

2. CPSD in ADS and Monte Carlo simulations

This section reviews the formula representing the CPSD in an ADS into which periodically pulsed multiple spallation neutrons are injected. The formula is to be compared with the results of Monte Carlo simulations for noise measurements in the ADS. To eliminate the effects of spatial- and energy-higher order modes, an infinite homogeneous one-energy group multiplying system is considered. Suppose that q spallation neutrons are emitted at a constant period T. The source intensity (i.e., neutrons emitted at a time t) is given by

$$S(t) = q\sum_{m=-\infty}^{\infty}\delta(t-mT), \tag{1}$$

where $\delta()$ is the Dirac delta function and m is an integer. For this highly-idealized situation, the formula of the CPSD is presented in (Sakon et al., 2013). In the present paper, the formula is modified as

$$CPSD(\omega) = \varepsilon_1\varepsilon_2\frac{\overline{\nu(\nu-1)}}{L_f}\frac{q}{\alpha T(\alpha^2+\omega^2)} - \varepsilon_1\varepsilon_2\frac{q}{T(\alpha^2+\omega^2)} + \varepsilon_1\varepsilon_2\frac{q^2}{T^2}\sum_{m=-\infty}^{\infty}\frac{2\pi\delta(\omega-\omega_m)}{\alpha^2+\omega_m^2}, \tag{2}$$

where

$$\omega_m = \frac{2\pi m}{T}, \tag{3}$$

ω is the angular frequency, ε_i (i = 1 or 2) the detection efficiency of the detector i, α the prompt neutron time decay constant, ν the number of neutrons released per fission, L_f (=$1/(\upsilon\Sigma_f)$) the mean life time of a neuron for fission, υ the neutron velocity, and Σ_f is the macroscopic fission cross section. The first term on the right-hand side of Eq. (2) represents a correlated component that stems from the release of multiple neutrons from one fission event. The third term represents an uncorrelated component that appears at the integer multiples of the pulse repetition frequency ($1/T$). The second term is newly introduced in the present paper. This term stems from the correlation between multiple neutrons emitted from one spallation event. The details in Eq. (2) are discussed in the Appendix A. The measured CPSD represents the summation of both two components. The first term can be easily separated from the measured CPSD as:

$$CPSD^C(\omega) = \varepsilon_1\varepsilon_2\frac{\overline{\nu(\nu-1)}}{L_f}\frac{q}{\alpha T(\alpha^2+\omega^2)} - \varepsilon_1\varepsilon_2\frac{q}{T(\alpha^2+\omega^2)} \quad \text{for } \omega \neq \frac{2\pi m}{T}. \tag{4}$$

Both two equations, Eqs. (2) and (4), exhibit the same dependence on the frequency. Thus, either of Eqs. (2) and (4) is usable to obtain the prompt neutron time decay constant α by fitting to the function $A/(\alpha^2+\omega^2)$ where A is an arbitrary constant.

In this paper, conventional time domain Monte Carlo simulations were performed for an infinite homogeneous multiplying system. The parameters used for the simulations are shown in Table 1. Since ν, the number of neutrons per fission, is 2, the Diven-factor, $\nu(\nu-1)$, is 2. The simulations were performed with the analog Monte Carlo technique. All variance reduction techniques such as the weighting scheme, implicit capture, Russian roulette game, and splitting were disabled although recent works indicate that the variance reduction techniques are applicable to the Monte Carlo simulation of a reactor noise measurement (Szieberth and Kloosterman, 2010; Yamamoto, 2011a). The particle weights were consistently kept unity from birth to death. When a particle was killed by a capture reaction, the reaction was registered as one neutron count.

The calculation flow of the Monte Carlo simulations is shown as follows.

(1) Before starting the random walk processes, a data block is prepared. The data block is time samples of detector response composed of $M = 2^N$ time bins. Each time bin has a width of Δ and contains the number of detections that occur during the corresponding time bin.
(2) A spallation neutron is emitted at $t = t_0$, which coincides with the starting time of the date block.
(3) The neutron flies to the next collision site. The time of the neutron is updated to $t' = t - \ln\xi_1/(\upsilon\Sigma_t)$ where ξ_1 is a uniform pseudo random number from (0,1].
(4) If $\xi_2 < \Sigma_c/\Sigma_t$, the neutron is captured and the random walk is terminated. One neutron count is registered in the time bin corresponding to the time t'. To discriminate two detectors, the count is sorted into the detector 1 or 2 according to whether ξ_3 is larger than 0.5 or not. Thus, the detector efficiencies are $\varepsilon_1 = \varepsilon_2 = \upsilon\Sigma_c/2$.
(5) If the reaction is not a capture and $\xi_4 < \Sigma_f/\Sigma_t$, the neutron induces a fission reaction and the random walk is terminated. Int($\nu + \xi_5$)neutrons are stored in the fission source

Table 1
Parameters used for Monte Carlo simulations.

υ (cm/s)	2.2×10^5
ν	2
Σ_t (cm^{-1}) (total cross section)	0.2
Σ_f (cm^{-1}) (fission cross section)	0.048
Σ_c (cm^{-1}) (capture cross section)	0.05
Σ_s (cm^{-1}) (scattering cross section)	0.102
k_∞	0.97959
α (s^{-1})	440