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Performance evaluation of multi-state degraded systems with minimal repairs and imperfect preventive maintenance

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ABSTRACT

In this paper, we develop a model for evaluating the availability, the production rate and the reliability function of multi-state degraded systems subjected to minimal repairs and imperfect preventive maintenance. The status of the system is considered to degrade with use. These degradations may lead to decrease in the system efficiency. It is assumed that the system can consecutively degrade into several discrete states, which are characterized by different performance rates, ranging from perfect functioning to complete failure. The latter is observed when the degradation level reaches a certain critical threshold such as the system efficiency may decrease to an unacceptable limit. In addition, the system can fail randomly from any operational or acceptable state and can be repaired. This repair action brings the system to its previous operational state without affecting its failure rate (i.e., minimal repair). The used preventive maintenance policy suggests that if the system reaches the last acceptable degraded state, it is brought back to one of the states with higher efficiency. Considering customer demand as constant, the system is modeled as a continuous-time Markov process to assess its instantaneous and stationary performance measures. A numerical example is given to illustrate the proposed model.

1. Introduction

In binary reliability modeling, the system is assumed to be either in a working state or in a failed one. However, in many real-life situations, this binary-state assumption may not be adequate. In multi-state reliability modeling, the system may rather have more than two levels of performance varying from perfect functioning to complete failure. A multi-state system (MSS) may perform at different intermediate states between working perfectly and total failure. The presence of degradation is a common situation in which a system should be considered to be an MSS. Degradation can be caused by system deterioration or by variable ambient conditions. Fatigue, failures of non-essential components, and number of random shocks on the system are all examples of system degradation causes. In this case, the failure rate depends on the status of the system which can degrade gradually. The reliability analysis of such degraded systems should consider multiple operational states to take into account multiple degradation levels.

The basic concepts of MSS reliability were first introduced in [1–4]. These works defined the system structure function and its properties. They also introduced the notions of minimal cut set and minimal path set in MSS context, and studied the notions of coherence and component relevancy. A literature review on MSS reliability can be found, for example, in Ref. [5]. The methods currently used for MSS reliability estimation are generally based on four different approaches: (i) the structure function approach, which extends Boolean models to the multi-valued case (e.g., [2–4]), (ii) the Monte-Carlo simulation technique (e.g., [6]), (iii) the Markov process approach (e.g., [7,8]), and (iv) the universal moment generating function (UMGF) method (e.g., [9,10]). These approaches are often used by practitioners, for example, in the field of power systems reliability analysis [5,11]. In practice, different reliability measures can be considered for MSS evaluation and design [12,13]. For example, the availability of a repairable MSS is defined by the system ability to meet a customer's demand (required performance level). In power systems, it is the ability to provide an adequate supply of electrical energy [11].

To improve the performance of a multi-state degraded system, preventive maintenance (PM) plays a key role. Perfect PM is aimed at making the MSS 'as good as new', while imperfect PM may bring the MSS back to an intermediate state between the current state and the perfect functioning state. In Ref. [14], the authors study a deteriorating repairable MSS with an imperfect PM policy that is based on the failure number of the system. In Ref. [15], a model of MSS with state-dependent cost is considered. The state

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space of the system is partitioned into two subsets: the first represents all states of normal operations, while the second represents the single failure state. A periodic maintenance model is developed and the optimal cycle time of maintenance actions is determined over a specific finite horizon. More recently, in Ref. [16] the author develops a monotone process maintenance model for an MSS. A replacement policy that is based on the failure number of the system is studied. An analytical approach is used to determine the optimal replacement policy.

Although PM policies have received extensive interest in the context of binary-state systems, less attention has been paid to imperfect PM for MSS. In this paper, we deal with the performance evaluation of a multi-state degraded system subjected to imperfect preventive maintenance and minimal repairs. Even if real systems may degrade continuously, we will consider that an MSS degrades consecutively into a finite number of discrete states. This discrete approximation is motivated not only by the complexity of continuous models, but also by the normal functioning of MSS. We model the considered system as a Markov chain and we evaluate three of its performance measures, namely the availability, the production rate, and the reliability function.

The remainder of the paper is organised as follows. In Section 2, we present the assumptions and we describe the multi-state degraded system under study. In Section 3, we present our approach to evaluate the performance measures of multi-state degraded systems subjected to imperfect preventive maintenance and minimal repairs. In Section 4, we provide a numerical example to illustrate the model. Finally, a conclusion is given in Section 5.

2. Model description

2.1. Assumptions

(1) The system may have many levels of degradation, corresponding to discrete performance rates, which vary from perfect functioning to complete failure.
(2) The system might fail randomly from any operational state (i.e., from the perfect functioning as well as from any degraded acceptable state) and it is minimally repaired.
(3) All transition rates are constant and exponentially distributed.
(4) The current degradation state is observable through some system parameter(s), and the time needed for inspection is negligible (i.e., inspection is instantaneous).

2.2. System description

The system is initially in its perfect functioning state (also called nominal state or good state). As time progresses, it can either go to the first degraded state upon degradation, or it can go to a failed state upon a *random and sudden* failure called *Poisson failure*. Such a Poisson failure occurs abruptly unlike the gradually worsening deterioration failures. If the system fails after a Poisson failure, it is minimally repaired. When a system reaches the first degraded state, it can either go to the second degraded state upon degradation or can go to a failed state from which a minimal repair is performed. The same process will continue for all acceptable degraded states. When the system reaches an unacceptable state, it cannot satisfy the demand (required performance level), and this must be treated as a failure. If the inspection finds the system in its last acceptable state, a preventive maintenance (PM) is performed to restore the system to one of the previous higher performance states. Several kinds of PM actions are possible, varying from minor maintenance to major maintenance. A minor maintenance restores the system to the previous degraded state, while a major maintenance restores it to the "as good as new" state (i.e., the initial perfect functioning state). Fig. 1 shows the system–state transition diagram using the following notations:

(i): State (i); $i=1, \ldots, n$ and $n=2d+m$.

State (1): Perfect functioning.
State $(2j-1)$: Degraded; $j=2, \ldots, d$.
State $(2j)$: Failed from an operational state; $j=1, \ldots, d$.
State $(2d+k)$: Failed after a degradation process; $k=1, \ldots, m$.
λ_j: Failure rate or transition rate from state $(2j-1)$ to state $(2j)$; $j=1, \ldots, d$.
μ_j: Minimal repair rate or transition rate from state $(2j)$ to state $(2j-1)$; $j=1, \ldots, d$.
α_j: Degradation rate or transition rate from state $(2j-1)$ to state $(2j+1)$; $j=1, \ldots, d$.
β_j: Transition rate from state $(2d-1)$ to state $(2j-1)$; $j=1, \ldots, d-1$.

Note that any specific system evaluation presumes existence of just one PM transition from state $(2d-1)$ to *only one* of the degraded states $(2j-1)$, $j=2, \ldots, d$. For example, if the chosen PM action is a major maintenance, the only transition rate used from state $(2d-1)$ is β_1.

3. Evaluation of performance indices

3.1. Classification of the states

Each state $(2j-1)$, $j=1, \ldots, d$, in Fig. 1 is characterized by a level of efficiency or a performance rate denoted by G_{2j-1}, ranging from the best performance rate G_1 to the last acceptable one G_{2d-1} $(G_1 > G_3 > G_{2d-1})$. Such a system is called a multi-state system because it can have a finite number of performance rates [12]. The performance rate of the failed states is zero (i.e., $G_{2j}=0$, $j=1, \ldots, d$). The states $(2d+k)$, $k=1, \ldots, m$, have performance rates ranging from G_{2d+1} for state $(2d+1)$ to 0 for the last state

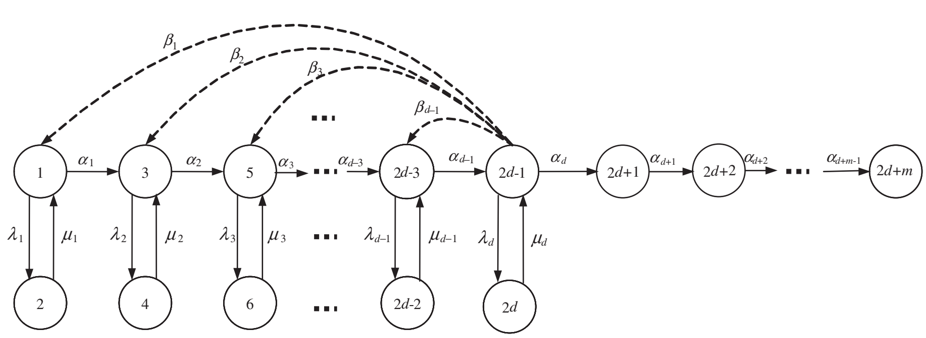

Fig. 1. System state transition diagram.