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Heterogeneous reactor core transport technique using response matrix and collision probability methods

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ABSTRACT

In this paper a whole-core transport technique using response matrix and collision probability (CP) methods is presented for large-scale, highly-heterogeneous reactors. Integral transport method has been used to provide sufficient accuracy for response matrix formation of pin cell sized node with considerably less computational expense. Ray tracing is efficiently applied using macro-bands. For practical application, double P_2 (DP_2) Legendre polynomial expansion is applied to approximate interface angular flux that is used to couple nodes. The proposed method is based on a sound mathematical foundation and leads to dramatically reduced memory requirements in contrast to the conventional transport method. This method is also applied to several problems such as C5G7, containing mixed oxide (MOX) and UO_2 fuel assemblies, to show the effectiveness of the proposed method. The results clearly indicate that the method is quite promising and acceptable.

1. Introduction

Reactor core analysis is a dominant procedure leads to economic and safe operation of nuclear power plants. At present, whole-core analysis of reactors is accomplished primarily through a series of detailed transport calculations for each lattice cell in infinite medium to produce spatially homogenized and energy collapsed lattice cross-sections and flux discontinuity factors (Smith, 1986). These parameters are finally used in a few-group coarse mesh diffusion methods (Lawrence, 1986). The fuel pin powers are subsequently calculated using reconstruction techniques. The corresponding approximation procedures have been widely investigated and prosperously applied to analysis of reactors.

With increasing heterogeneity of reactor cores, the limitations due to homogenization procedure, such as infinite-medium boundary condition, and diffusion theory, which breaks down because of large flux gradients, are evident. In recent years, increasing efforts have been devoted to overcome the restriction through using more highly sophisticated homogenization procedure or whole-core transport calculations.

To achieve equivalence cross-section in highly heterogeneous media, a high-order homogenization method based on high-order boundary condition perturbation theory is proposed which can consider cross-sections and discontinuity factors variation (Rahnema and McKinley, 2002). Clarno and Adams (2005) proposed that the effect of unlike neighboring assemblies can be captured from four-assembly calculations. They used a superposition technique to reduce the number of calculations.

Whole-core transport calculations can be considered as a confident solution to the uncertainties imposed by homogenization-reconstruction techniques. To address these complicated and vital issues, CRX had been constructed to treat two-dimensional geometries based on the method of characteristics and specific features, such as modular ray tracing and parallel computation (Hong and Cho, 1998). De-CART was also developed for direct whole-core calculation that utilized the planar method of characteristics plus coarse mesh finite difference method (Cho and Joo, 2006; Joo et al., 2004). However, these transport methods introduce a high computational cost. Therefore, Mosher and Rahnema (2006) developed an incident flux response expansion method for heterogeneous coarse mesh transport problems. In this approach, the response function of unique coarse mesh is generated and coupled using expanded angular flux at the mesh boundaries (Forget et al., 2004). In progress of finite element method, VARIANT (Carrico et al., 1992; Palmiotti et al., 1995) was introduced to employ the finite subelement form of the variational nodal transport method. In this approach, heterogeneous response matrices are formed at the pin cell level, and spherical harmonics treat the angular variables at the node interfaces (Smith et al., 2004; Smith et al., 2003). Villarino and Stamm'ler (1984) also developed a heterogeneous coarse mesh method that response matrices were computed using the CP techniques. The coarse meshes were coupled by cosine interface currents in slab geometry, and accurate results were produced.

The CP method is an accurate and versatile transport method (Sanchez and McCormick, 1982) that was applied to cell

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calculation codes such WIMS (Askew et al., 1966) and CPM (Ahlin and Edenius, 1977). The cosine and double P_1 (DP_1) Legendre expansions of angular flux had been developed and applied in two-dimensional fuel assembly cell calculation codes such as CASMO (Edenius et al., 1990; Häggblom et al., 1975), PHOENIX (Weiss and Stamm'ler, 1977), and APOLLO (Kavenoky and Sanchez, 1987).

In this paper, we developed a new heterogeneous whole-core transport method. The response matrices have been generated using the CP method at the level of the pin cell. To couple the pin cells, DP_2 Legendre polynomial expansion is used, and Carlvik's method is employed to accurately and efficiently evaluate the CPs (Carlvik, 1967).

The remainder of the paper is organized as follows. In Section 2, integral particle transport equation is presented. Section 3 discusses the concept of response matrix formation and its derivation using integral transport equation. In Section 4, the CPs are derived and presented. Section 5 represents expression of response matrices in term of the CPs. The ray tracing procedure is discussed in Section 6. Section 7 presents benchmark results obtained by developed computer program HERPAT. Finally, conclusion and remarks are given in Section 8.

2. Integral particle transport

The integral particle transport equation is obtained through rewriting the streaming operator, $\Omega\cdot\nabla$, of the Boltzmann particle transport equation to the directional derivative, d/dR, along the particle flight path. The derivative is removed through using the integration factor and integrating along the particle travel (Lewis and Miller, 1993). Therefore, the two-dimensional multi-group integral particle transport equation can be written as

$$\phi^g(r,\Omega) = \phi^g_-(r_{s'},\Omega)e^{-\frac{\tau^g(r,r_{s'})}{\sin\theta}} + \int_0^{R_{s'}} dR'\frac{Q^g(r',\Omega)}{\sin\theta}e^{-\frac{\tau^g(r,r')}{\sin\theta}} \quad (1)$$

where the source is defined as

$$Q^g(r,\Omega) = \sum_{g'=1}^{G}\int_{4\pi} d\Omega'\Sigma_s^{g'\to g}(r,\Omega' \to \Omega)\phi^{g'}(r,\Omega') + \frac{\chi^g}{k}\sum_{g'=1}^{G}\nu\Sigma_f^{g'}(r)\phi^{g'}(r,\Omega) + Q^g_{sc}(r,\Omega) \quad (2)$$

suppressing the energy group index, g, $\phi(r,\Omega)$ is the angular flux at location r and direction Ω; ϕ_- is the incoming angular flux; Q_{sc} is the fixed source; τ is the optical distance; $\Sigma_s(r, \Omega' \to \Omega)$ is the scattering cross-section from Ω' to Ω; $\nu\Sigma_f$ is the nu-fission cross-section; k is the effective multiplication factor; χ is the fission spectrum and θ is polar angle.

As depicted in Fig. 1, the flux is obtained through summing up the uncollided particles at location $r_{s'}$ in surface s and integration of the uncollided particles that are generated along the direction Ω at location r' in volume V.

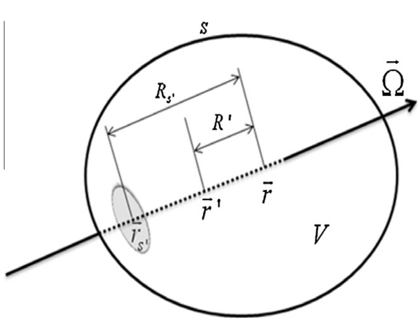

Fig. 1. Coordinate characterizing particle transport.

The outgoing angular flux can be obtained as:

$$\phi^g_+(r_s,\Omega) = \phi^g_-(r_{s'},\Omega)e^{-\frac{\tau^g(r_s,r_{s'})}{\sin\theta}} + \int_0^{R_{s'}} dR'\frac{Q^g(r',\Omega)}{\sin\theta}e^{-\frac{\tau^g(r_s,r')}{\sin\theta}} \quad (3)$$

The boundary condition is considered as:

$$\phi^g_-(r_s,\Omega) = \int_{n_+\cdot\Omega'>0} d\Omega'\phi^g_+(r_{s'},\Omega')T(r_{s'},\Omega'\to\Omega) \quad (4)$$

with T denoting the reflection and transmission operator and n_+ is outward normal vector on the surface.

3. Response matrix formulation

The core of the reactor is partitioned into heterogeneous space elements consisting of regions and bounding surface segments. The space elements are coupled together by requiring continuity of the partial currents across the nodal interfaces. The responses of each space element should be obtained for determining flux of the regions and the partial currents across the segments of the node (Lindahl and Weiss, 1981; Forget et al., 2004).

The responses can be classified into four categories as generation, transmission, entrance and exit. The generation response matrix consists of the contribution of the region flux from the generated particle in the entire regions, the transmission response matrix tackles the contribution of the incoming angular flux moment to the outgoing current moment, the exit and the entrance response matrices deal with the contribution of the generated particle in the entire regions to the outgoing angular flux moment and the contribution of the entered particle to flux of the region, respectively.

In this paper, for developing the response matrices, the following spherical harmonic expansion is used:

$$\phi^g_\pm(r_s,\Omega) = \sum_{l=0}^{2}\sum_{m=-l}^{l}\frac{2l+1}{2\pi}\phi^g_{\pm lm}(r_s)\psi_{lm}(\theta,\omega) \quad (5)$$

where $\phi^g_{\pm lm}$ is the moment lm of the segment angular flux in group g, and ± represents the outward or the inward particles and ψ_{lm} is the orthogonal function:

$$\psi_{lm}(\theta,\omega) = \left[(2-\delta_{m0})\frac{(l-m)!}{(l+m)!}\right]^{1/2}P_{lm}(\cos\theta)e^{im2\omega} \quad (6)$$

with θ denoting polar angle; ω is azimuthal angle; δ_{m0} is Kronecker delta function and P_{lm} is associated Legendre polynomial. The orthogonality relation of the half-range spherical harmonics is:

$$\int_{n_\pm\cdot\Omega>0}\psi_{lm}(\theta,\omega)\psi_{l'm'}(\theta,\omega)d\Omega = \frac{2\pi}{2l+1}\delta_{ll'}\delta_{mm'} \quad (7)$$

with n denoting normal vector on the surface and ± represents the outward or the inward particles at the surface segment.

As shown in Fig. 2, the space element is divided into homogeneous regions that the flux is approximated to be flat. The surface is also divided into segments that the current is assumed to be flat. Moreover, the source assumed to be isotropic. Therefore, the following equations can be obtained by integrating Eq. (1) over angle Ω and the volume and Eqs. (3) and (4) over angle Ω and the surface:

$$\boldsymbol{\phi}^g = \sum_{l=0}^{2}\sum_{m=-l}^{l}\boldsymbol{\phi}^g_{-lm}\boldsymbol{Y}^g_{lm} + \boldsymbol{Q}^g\boldsymbol{X}^g \quad (8)$$

$$\boldsymbol{J}^g_{+l'm'} = \sum_{l=0}^{2}\sum_{m=-l}^{l}\boldsymbol{\phi}^g_{-lm}\boldsymbol{P}^g_{l'm',lm} + \boldsymbol{Q}^g\boldsymbol{P}^g_{l'm'} \quad (9)$$