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Non probabilistic solution of uncertain neutron diffusion equation for imprecisely defined homogeneous bare reactor

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ABSTRACT

The scattering of neutron collision inside a reactor depends upon geometry of the reactor, diffusion coefficient and absorption coefficient etc. In general these parameters are not crisp and hence we get uncertain neutron diffusion equation. In this paper we have investigated the above equation for a bare square homogeneous reactor. Here the uncertain governing differential equation is modelled by a modified fuzzy finite element method. Using modified fuzzy finite element method, obtained eigenvalues and effective multiplication factors are studied. Corresponding results are compared with the classical finite element method in special cases and various uncertain results have been discussed.

1. Introduction

Uncertainty plays a vital role in various fields of engineering and science. These uncertainties occur due to incomplete data, impreciseness, vagueness, experimental error and different operating conditions influenced by the system. Different authors proposed various methods to handle uncertainty. They have used probabilistic or statistical method as a tool to handle the uncertain parameters. In this Context Monte Carlo method is an alternate method which is based on the statistical simulation of the random numbers generated on the basis of a specific sampling distribution. Monte Carlo methods have been used to solve the neutron diffusion equation with variable parameters. As such, Nagaya et al. (2010) implemented Monte Carlo method to estimate the effective delayed neutron fraction β_{eff}. Further, Nagaya and Mori (2011) proposed a new method to estimate the effective delayed neutron fraction β_{eff} in Monte Carlo calculations. In the above paper, the eigenvalue method is jointly used with the differential operator and correlated sampling techniques, whereas, Shi and Petrovic (2011) used Monte Carlo methods to solve one-dimensional two-group problems and then they proved its validity for these problems. Sjenitzer and Hoogenboom (2011) gave an analytical procedure to compute the variance of the neutron flux in a simple model of a fixed-source calculation. Recently, Yamamoto (2012) investigated the neutron leakage effect specified by buckling to generate group constants for use in reactor core designs using Monte Carlo method.

As such in the above process we need a good number of observed data or experimental results to analyse the problem. Sometimes it may not be possible to get a large number of data. As regards; Zadeh (1965) proposed an alternate idea viz. fuzzy approach to handle uncertain and imprecise variables. Accordingly, we may use interval or fuzzy parameters to take care of the uncertainty. In general traditional interval/fuzzy arithmetic are complicated to investigate the problem. As regards, we have proposed a new technique for fuzzy arithmetic to overcome such difficulty. The idea as proposed by Chakraverty and Nayak (2012) has been extended. Few authors have investigated the said problem. In this respect, Biswas et al. (1976) have given a method of generating stiffness matrices for the solution of multi group diffusion equation by natural coordinate system. Azekura (1980) has also proposed a new representation of finite element solution technique for neutron diffusion equations. The author has applied the technique to two types of one-group neutron diffusion equations to test its accuracy. Further, Cavdar and Ozgener (2004) developed a finite element–boundary element hybrid method for one or two group neutron diffusion calculations. In their paper linear or bilinear finite element formulation for the reactor core and a linear boundary element technique for the reflector which are combined through interface continuity conditions constitute the basis of the developed method. Saed et al. (2011) formulated an alternative analytical solution of the neutron diffusion equation for both infinite and finite cylinders of fissile material using the homotopy perturbation method. Whereas, Rokrok et al. (2012) applied Element-Free Galerkin (EFG) method to solve the neutron diffusion equation in X–Y

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geometry. It reveals from the above literature that the neutron diffusion equations are solved using finite element method in presence of crisp parameters only.

But the presence of uncertain parameters makes the system uncertain and we get uncertain governing differential equations. To the best of our knowledge, no study has been done for fuzzy/interval uncertain neutron diffusion equation. In this context, uncertain fuzzy parameters are considered to solve heat conduction problems using finite element method and we call it as fuzzy finite element method (FFEM). Bart et al. (2011) solved the uncertain solution of heat conduction problem. In this paper authors gave a good comparison between response surface method and other methods. Recently, Chakraverty and Nayak (2012) also solved the interval/fuzzy distribution of temperature along a cylindrical rod.

In view of the above, here we present a modified form of fuzzy finite element method. The involved fuzzy numbers are changed into intervals through α-cut. Then the intervals are transformed into crisp form by using some transformations. Crisp representations of intervals are defined by symbolic parameterization. Traditional interval arithmetic is modified using the crisp representation of intervals. The proposed interval arithmetic is extended for fuzzy numbers and the developed fuzzy arithmetic is used as a tool for uncertain fuzzy finite element method. Consequently the above method is used to solve one group neutron diffusion equation and the critical eigenvalues and effective multiplication factors are studied in detail. Finally some important conclusions of the proposed methods are encrypted and it is found that this method is simpler and efficient to handle. Hence it may be used as a tool to solve different types of neutron diffusion problems for various types of nuclear reactors.

2. Interval and fuzzy arithmetic

The uncertain values occurred in practical cases (such as experimental data, impreciseness and partial or imperfect knowledge) may be handled by taking the uncertainty as interval or as fuzzy sense. So to compute these uncertainties we need interval/fuzzy arithmetic. Let us consider the uncertain values in interval form and the same may be written in the following way.

$[\underline{x},\bar{x}] = \{x | x \in R, \underline{x} \leqslant x \leqslant \bar{x}\}$

where $\underline{x}$ and $\bar{x}$ are lower and upper values of the interval respectively. Let us consider $m = \frac{\underline{x}+\bar{x}}{2}$ and $w = \bar{x} - \underline{x}$ are mid or centre and the width of the interval $[\underline{x},\bar{x}]$ respectively.

Let us assume that $[\underline{x},\bar{x}]$ and $[\underline{y},\bar{y}]$ be two intervals then

1. $[\underline{x},\bar{x}] + [\underline{y},\bar{y}] = [\underline{x}+\underline{y}, \bar{x}+\bar{y}]$.
2. $[\underline{x},\bar{x}] - [\underline{y},\bar{y}] = [\underline{x}-\bar{y}, \bar{x}-\underline{y}]$.
3. $[\underline{x},\bar{x}] \times [\underline{y},\bar{y}] = [\min\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}]$.
4. $[\underline{x},\bar{x}] \div [\underline{y},\bar{y}] = [\min\{\underline{x} \div \underline{y}, \underline{x} \div \bar{y}, \bar{x} \div \underline{y}, \bar{x} \div \bar{y}\}, \max\{\underline{x} \div \underline{y}, \underline{x} \div \bar{y}, \bar{x} \div \underline{y}, \bar{x} \div \bar{y}\}]$.

We may extend this concept into various fuzzy numbers viz. triangular and trapezoidal fuzzy numbers, etc. Any arbitrary fuzzy number may be defined in terms of interval involving left and right continuous linear functions. Fuzzy numbers may be represented as an ordered pair form $[\underline{f}(\alpha), \bar{f}(\alpha)]$, $0 \leqslant \alpha \leqslant 1$ where $\underline{f}(a)$ and $\bar{f}(\alpha)$ are left and right monotonic increasing and decreasing functions over $[0,1]$ respectively.

Let us consider two fuzzy numbers $x = [\underline{x}(\alpha), \bar{x}(\alpha)]$ and $y = [\underline{y}(\alpha), \bar{y}(\alpha)]$ and a scalar k then

i. $x = y$ If and only if $\underline{x}(\alpha) = \underline{y}(\alpha)$ and $\bar{x}(\alpha) = \bar{y}(\alpha)$.
ii. $x + y = [\underline{x}(\alpha) + \underline{y}(\alpha), \bar{x}(\alpha) + \bar{y}(\alpha)]$.
iii. $kx = \begin{cases} [k\underline{x}(\alpha), k\bar{x}(\alpha)], & k \geqslant 0, \\ [k\bar{x}(\alpha), k\underline{x}(\alpha)], & k < 0 \end{cases}$.

Definition 2.1. Above interval arithmetic for real interval are defined here as follows.

1. $[\underline{x},\bar{x}] + [\underline{y},\bar{y}] = [\min\{\lim_{n\to\infty} l_1 + \lim_{n\to\infty} l_2, \lim_{n\to 1} l_1 + \lim_{n\to 1} l_2\}, \max\{\lim_{n\to\infty} l_1 + \lim_{n\to\infty} l_2, \lim_{n\to 1} l_1 + \lim_{n\to 1} l_2\}]$.
2. $[\underline{x},\bar{x}] - [\underline{y},\bar{y}] = [min\{lim_{n\to\infty} l_1 - lim_{n\to 1} l_2, lim_{n\to 1} l_1 - lim_{n\to\infty} l_2\}, max\{lim_{n\to\infty} l_1 - lim_{n\to 1} l_2, lim_{n\to 1} l_1 - lim_{n\to\infty} l_2\}]$.
3. $[\underline{x},\bar{x}] \times [\underline{y},\bar{y}] = [min\{lim_{n\to\infty} l_1 \times lim_{n\to\infty} l_2, lim_{n\to 1} l_1 \times lim_{n\to 1} l_2\}, max\{lim_{n\to\infty} l_1 \times lim_{n\to\infty} l_2, lim_{n\to 1} l_1 \times lim_{n\to 1} l_2\}]$.
4. $[\underline{x},\bar{x}] \div [\underline{y},\bar{y}] = [\min\{\lim_{n\to\infty} l_1 \div \lim_{n\to 1} l_2, \lim_{n\to 1} l_1 \div \lim_{n\to\infty} l_2\}, \max\{\lim_{n\to\infty} l_1 \div \lim_{n\to 1} l_2, \lim_{n\to 1} l_1 \div \lim_{n\to\infty} l_2\}]$.

where for an arbitrary interval $[\underline{a},\bar{a}] = \{\underline{a} + \frac{w}{n} = l | \underline{a} \leqslant l \leqslant \bar{a}, n \in [1,\infty)\}$ and $w = \bar{a} - \underline{a}$ is the width of the interval.

Definition 2.2. A fuzzy number $\widetilde{A} = [a^L, a^N, a^R]$ is said to be triangular fuzzy number (Fig. 1) when the membership function is given by

$$\mu_{A^\sim}(x) = \begin{cases} 0, & x \leqslant a^L; \\ \frac{x-a^L}{a^N-a^L}, & a^L \leqslant x \leqslant a^N; \\ \frac{a^R-x}{a^R-a^N}, & a^N \leqslant x \leqslant a^R; \\ 0, & x \geqslant a^R. \end{cases}$$

Definition 2.3. A fuzzy number $\widetilde{A} = [a^L, a^{NL}, a^{NR}, a^R]$ is said to be trapezoidal fuzzy number (Fig. 2) when the membership function is given by

$$\mu_{A^\sim}(x) = \begin{cases} 0, & x \leqslant a^L; \\ \frac{x-a^L}{a^{NL}-a^L}, & a^L \leqslant x \leqslant a^{NL}; \\ 1, & a^{NL} \leqslant x \leqslant a^{NR}; \\ \frac{a^R-x}{a^R-a^{NR}}, & a^{NR} \leqslant x \leqslant a^R; \\ 0, & x \geqslant a^R. \end{cases}$$

Definition 2.4. The triangular fuzzy number $\widetilde{A} = [a^L, a^N, a^R]$ and trapezoidal fuzzy number $\widetilde{B} = [a^L, a^{NL}, a^{NR}, a^R]$ may be transformed into interval form by using α-cut in the following form

$$\widetilde{A} = [a^L, a^N, a^R] = [a^L + (a^N - a^L)\alpha, a^R - (a^R - a^N)\alpha],$$
$$\widetilde{B} = [a^L, a^{NL}, a^{NR}, a^R] = [a^L + (a^{NL} - a^L)\alpha, a^R - (a^R - a^{NR})\alpha], \quad \alpha \in [0,1].$$

Definition 2.5. If the fuzzy numbers are taken in interval form then using Definition 2.1 the arithmetic rules may be defined as

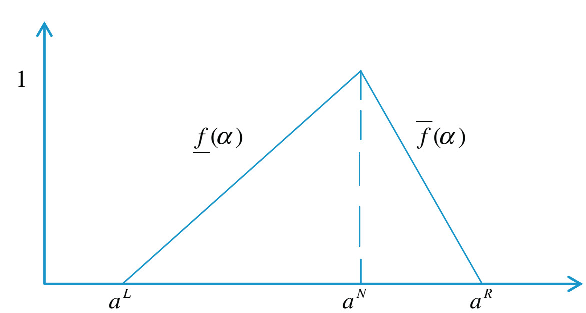

Fig. 1. Triangular fuzzy number (TFN).