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Regression analysis of the structure function for reliability evaluation of continuous-state system

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ABSTRACT

Technical systems are designed to perform an intended task with an admissible range of efficiency. According to this idea, it is permissible that the system runs among different levels of performance, in addition to complete failure and the perfect functioning one. As a consequence, reliability theory has evolved from binary-state systems to the most general case of continuous-state system, in which the state of the system changes over time through some interval on the real number line. In this context, obtaining an expression for the structure function becomes difficult, compared to the discrete case, with difficulty increasing as the number of components of the system increases. In this work, we propose a method to build a structure function for a continuum system by using multivariate nonparametric regression techniques, in which certain analytical restrictions on the variable of interest must be taken into account. Once the structure function is obtained, some reliability indices of the system are estimated. We illustrate our method via several numerical examples.

1. Introduction

Most research in reliability models have traditionally concentrated in a binary formulation of systems behaviour, that is, models allow only two states of functioning for a system and its components: perfect functioning and complete failure. However, in practice, many systems may experience continuous degradation so that they can exhibit different levels of performance between the two extreme cases of full functioning and fatal failure. A typical example is a system subject to wear, which degrades continuously with time, so its performance properties decrease progressively and, as consequence, it is necessary to consider a wider specification of the state space in order to have a more precise and appropriate description of the behaviour of the system at each time.

Baxter [1,2] first introduced continuum models for reliability systems, and, since then, a wide variety of performance measures have been defined and calculated to be valid for binary, multi-state and continuum systems (see [3] for an extensive review and more recently [4]). As a particular case, the structure function of the system, which represents the link function between system state and its components, has been a subject of primary interest in the field of reliability engineering. Since the reliability evaluation can be a very difficult problem in practice, even for relatively simple systems (see [5,6], for instance), it seems reasonable that to have a procedure for modelling the relationship between the system state and its components may assists efficiently in the reliability assessment of complex systems.

For binary systems, the structure function can be determined if the minimal paths or minimal cuts are known [7]. If it is the case of more than two states, several procedures have been developed in order to generalize the concept of binary coherent structure to a multi-state configuration, and so, the structure function can be specified via a finite set of boundary points, see El-Neweihi et al. [8] for a complete treatment of the problem. Later, Aven [9] justifies the introduction of multi-state models by the needs in some areas of application, such as gas/oil production and transportation systems, where a binary approach would give a poor representation of the real situation. He investigates the problem of computing relevance performance measures for a multi-state monotone system, some comparisons of the accuracy of such computations and the ones obtained by Monte Carlo methods are presented.

In [5] Boolean model is derived in order to describe the state of a multi-state system, revealing that the reliability evaluation of a system is a difficult task from a practical viewpoint, even for systems not excessively complexes. Meng [10] carries out a comparative study of two upper bounds for the reliability of a multi-state system, generalizing the binary case.

In case of a continuous system, if the structure function cannot be determined basing on qualitative characteristics (for instance, series or parallel structures) or boundary-point analysis, approximation methods are required. To that effect, several treatments of the problem have been carried out. Lisnianski [6] investigates an

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approach based on the universal generating function technique. The method consists of a discrete approximation of the continuous-state system performance by using a finite multi-state system and the purpose is to construct upper and lower bounds for the reliability measures of the continuous system.

Given the high difficulty inherent in the analytical evaluation of the performance of a continuous system, a new approach based on empirical methods has been introduced recently. Brunelle and Kapur [11] proposed a multivariate interpolation procedure by which a structure function for a continuous system is built starting from a scatterplot that represents the value of the state of the system at some values of the components values. Under this empirical perspective, we now propose a new technique that assumes a regression model for the structure function of a continuous system. The main purpose of this paper is to construct a structure function for the system given an observed set of states of the system and its components. For such a purpose, we propose the use of a class of monotone nonparametric regression techniques appropriate for the kind of problem we study here.

When the structure function Φ is monotone in one or more explanatory variables, it is appropriate to use any monotonic regression technique. Several numerical procedures are available from the specialized literature. Pool Adjacent Violators Algorithm (*PAVA*) is the method of most widespread use. It was first proposed to solve the one-dimensional case, when only one explanatory variable is considered, and it has been subsequently generalized to higher dimension problems (see [12]). Other solutions have been proposed to the problem of monotonic regression in one or more dimensions, [13,14] for instance, however we consider in this paper *PAVA* given that it can be easily implemented and extended to the case of multivariate regression. Moreover it can be applied after a nonparametric estimation method has been applied to a data set. In fact, our problem can be summarized as follows: *find a nonparametric monotone surface that fits properly a given data set*. In this aim, we explore a solution that combines local smoothing and isotonic regression.

The paper is organized as follows. Section 2 gives an overview of the problem of modelling the behaviour of continuum reliability structures that is our interest. Section 3, which is the main section of the paper, is devoted to explain the nonparametric regression techniques that we have adapted to the particular context of the structure function of a reliability system. The method demands to do two choices, i.e. the kernel function and the bandwidth parameter. So, in Section 3.3, we consider the problem of the boundary bias correction that is implicit in the case of compact support variables. In Section 3.4, we discuss the problem of selecting the bandwidth parameter by two different methods: the widely known and used Cross-Validation method and also a Bootstrap method. And finally, in Section 3.5, we expose the isotonic procedure that ensures the monotonicity required on the structure function of a coherent system and that is not guarantied, in principle, by the smooth estimation of the previous subsections. In Section 4, we carry out some simulation studies to illustrate the method. To finish, Section 5 gives some results in order to estimate some important measures of the system performance. Conclusions and further research are presented in Section 6. References section comprises some selected references on the subject.

2. Continuous structure function

Let Y be the random variable that denotes the state of a system composed by m elements, which are assumed mutually independent. The state of component i is a random variable X_i, for any $i=1, 2, \ldots, m$. We allow variables $X_1, X_2, \ldots, X_m$, and Y to take any value into the interval $[0,1]$. There is no loss of generality if we assume 0 as the worst state for the system as well as for any component (*complete failure*), and 1 is considered as the best state (*perfect functioning*).

For preliminary concepts, we establish the following notation.

2.1. Notation and definitions

m	the system size; i.e. the number of components
x_i	the state of component labelled i, $0 \leq x_i \leq 1$
$\mathbf{x}$	$(x_1, x_2, \ldots, x_m)$, a state-vector
$\mathbf{0}$	$(0, 0, \ldots, 0)$, *complete failure* state
$\mathbf{1}$	$(1, 1, \ldots, 1)$, *perfect functioning* state
$(\mathbf{x}\|v_i)$	$(x_1, x_2, \ldots, x_{i-1}, v, x_{i+1}, \ldots, x_m)$
X_i	the random variable for x_i
$\mathbf{X}$	$(X_1, X_2, \ldots, X_m)$
$\boldsymbol{S}$	$[0,1]^m$
$\prec$	the usual partial ordering defined in $\boldsymbol{S}$, i.e. $\mathbf{x}_1 \prec \mathbf{x}_2 \Leftrightarrow x_{1i} \leq x_{2i}$ for all $i=1, 2, \ldots, m$
y	the state of the system
Y	the random variable for y
$\Phi(\mathbf{x})$	the *structure function*. Then Φ: $[0,1]^m \to [0,1]$; and $y=\Phi(\mathbf{x})$
n	the sample size
i.i.d.	independent identically distributed
$\{(\mathbf{X}_j; Y_j) \in S \times [0,1]; j=1,2,\ldots,n\}$	a sample of *i.i.d.* $(m+1)$-dimensional observations
$s(\mathbf{x})$	a smooth estimate (not necessarily monotone) of $\Phi(\mathbf{x})$
$\phi(\mathbf{x})$	a monotone smooth estimate of $\Phi(\mathbf{x})$
K	a kernel function in the context of nonparametric estimation
h	a bandwidth parameter
ε	random perturbation
ω_i	a weighting function
Γ	the Gamma function
$\mathbf{A^t}$	transpose of matrix $\mathbf{A}$
∇	the gradient operator
$*$	indicates a bootstrap characteristic
$\mathbf{e^t}$	$(1, 0, \ldots, 0)$
$\tilde{K}(\cdot; a; b)$	density function of a Beta distribution with shape parameters a and b
μ	mean value of the Beta distribution
σ^2	variance of the Beta distribution
B	the bias of an estimator
V	the variance of an estimator
δ	a critical state of the system
#	number symbol
P_F	the probability of failure of the system
G	a grid of points in the unit interval
ASE	Averaged Squared Error
CV	Cross-Validation
FORM (*SORM*)	First (Second) Order Reliability Method
ISE	Integrated Squared Error
MLLS	Multivariate Local Linear Smoother
MNWS	Multivariate Nadaraya–Watson Smoother
MSE	Mean Squared Error
PAVA	Pool Adjacent Violators Algorithm
POF	probability of failure

2.2. Coherent systems

The structure function captures the relationships between the components of a system and the system itself, in such a way that the state of the system is known from the states of its components